Equal Gain Combining for Acquisition of Time-Hopping Ultra-Wideband Signals

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Abstract

The acquisition of ultra-wideband (UWB) signals is a potential bottleneck for system throughput in a packet-based network employing UWB signaling format in the physical layer. The problem is mainly due to the low received signal power which forces the acquisition system to process the signal over long periods of time before getting a reliable estimate of the timing of the signal. Hence there is a need to develop more efficient acquisition schemes by taking into account the signal and channel characteristics. In this paper, we investigate a single-user acquisition scheme which collects the energy in the multipaths by performing equal gain combining (EGC) to improve the acquisition performance. We define the *hit* set as the set of hypothesized phases which can guarantee adequate system performance after acquisition and also study the effect of the EGC window length on the acquisition performance.

Index Terms

Ultra-wideband, acquisition, equal-gain combining, multipath channels, serial search

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I. INTRODUCTION

Ultra-wideband (UWB) signaling [1], [2], [3] is under evaluation as a possible modulation scheme for wireless personal area network (PAN) protocols. The features of UWB radio which make it an attractive choice are its multiple access capabilities [1], [4], lack of significant multipath fading [5], [6], ability to support high data rates [7] and low transmitter power resulting in longer battery life for portable devices. The acquisition of the UWB signal is a potential bottleneck for system throughput in a packet-based network employing UWB signaling in the physical layer. The problem is mainly due to the low received signal power which forces the acquisition system to process the signal over long periods of time before getting a reliable estimate of the timing (phase) of the signal. Hence there is a need to develop more efficient acquisition schemes by taking into account the signal and channel characteristics. The UWB channel is a dense multipath channel without significant fading [6], [8]. In a dense multipath environment, there will be a considerable amount of energy available in the multipath components (MPCs). It seems reasonable to expect that an acquisition scheme which utilizes the energy in the MPCs would perform better than one which does not. Acquisition schemes which take into account the multipath nature of the channel have been developed for the case of DS-CDMA signals in [9], [10], [11], [12], while previous work on UWB acquisition [13], [14] has focussed on developing better search strategies.

In this paper, we propose a single-user acquisition scheme which performs equal gain combining (EGC) to collect the energy available in the MPCs resulting in faster acquisition. There are two main issues which arise if we adopt such a formulation. Firstly, in a multipath environment without significant fading the ambiguity function does not have an impulse-like shape as it does in the absence of multipaths. In fact, the ambiguity function decays slowly from its maximum value since a significant amount of energy is collected even if the hypothesized phase is not equal to the true phase but is sufficiently close to it. The fundamental difference between the acquisition problems in a multipath channel and in a channel without multipath is that there are more than one hypothesized phases which can be considered a *hit* or a good estimate of the true signal phase. Thus there is a need to redefine the set of hypothesized phases which correspond to a hit by taking into account the channel model and the length of the EGC window. Secondly, the choice of the length of the EGC window is not apparent. A small window will not collect enough energy and thus will result in a low probability of detecting the correct signal phase. A large window may collect a considerable amount of energy even when the true phase does not match the hypothesized phase, resulting in a high probability of false alarm. In this paper, we propose a definition of the set of hypothesized phases which correspond to a good estimate of the true signal phase by considering the system performance subsequent to acquisition. We also investigate the effect of the EGC window length on the mean acquisition time.

The paper is organized as follows. In Section II, we describe the system model under consideration. In Section III, we motivate and formulate EGC for acquisition in a multipath channel. In Section IV, we define the set of hypothesized phases which correspond to a *hit* by considering system performance subsequent to acquisition. We derive expressions for average probabilities of detection and false alarm in Section V. In Section VI we give a design criterion for choosing the decision threshold and study the effect of the EGC window length on the mean acquisition time for a serial search strategy. Section VII has the results followed by the conclusions in Section VIII.

II. System Model

A. Channel Model

We assume that the propogation channel is modeled by the UWB indoor channel model described in [15]. This model gives a statistical distribution for the path gains based on a UWB propagation experiment but does not address the issue of characterization of the received waveform shape. Due to the frequency sensitivity of the UWB channel, the pulse shapes received at different excess delays are path-dependent [16]. To enable tractable analysis, we assume that the pulse shapes associated with all the propagation paths are identical. The channel is then a stochastic tapped delay line model expressed as

$$h(t) = \sum_{k=0}^{N_{tap}-1} h_k f(t - kT_c), \tag{1}$$

where N_{tap} is the number of taps in the channel response, $T_c = 2$ ns is the tap spacing, h_k is the path gain at excess delay kT_c and $f(t)$ models the combined effect of the transmitting antenna and the propagation channel on the transmitted pulse. The path gains are independent but not identically distributed with a Nakagami-*m* distribution. The average energy gains $\Omega_k = E[h_k^2]$ of the path gains normalized to the total energy received at one meter distance are given by

$$\Omega_k = \begin{cases} \frac{E_{tot}}{1+rF(\epsilon)}, & \text{for } k = 0 \\ \frac{E_{tot}}{1+rF(\epsilon)} re^{-((k-1)T_c/\epsilon)}, & \text{for } k = 1, 2, \dots, N_{tap} - 1, \end{cases} \tag{2}$$

where E_{tot} is the total average energy in all the paths normalized to the total energy received at one meter distance, r is the ratio of the average energy of the second MPC and the average energy of the direct path, ϵ is the decay constant of the power delay profile and $F(\epsilon) = \frac{1-\exp[-(N_{tap}-1)kT_c/\epsilon]}{1-\exp(-kT_c/\epsilon)}$. According to [15], E_{tot}, r and ϵ are all modeled by lognormal distributions. The Nakagami fading figures $\{m_k\}$ are distributed according to truncated Gaussian distributions whose mean and variance vary linearly with excess delay. In this paper, these long-term statistics are treated as constants over the duration of the acquisition process.

B. Transmitted Signal

The transmitted signal is given by

$$x(t) = \sqrt{P} \sum_{l=-\infty}^{\infty} \psi(t - lT_f - c_l T_c), \tag{3}$$

where $\psi(t)$ is the UWB monocycle waveform, P is the transmitted power, T_f is the pulse repetition time, $\{c_l\}$ is the pseudorandom time-hopping (TH) sequence with period N_{th} taking integer values between 0 and $N_h - 1$, and T_c is the step size of the additional time shift provided by the TH sequence which is related to the pulse repetition rate as $T_f = N_h T_c$.

C. Received Signal

The received signal is given by

$$r(t) = \sqrt{P_r} \sum_{l=-\infty}^{\infty} w(t - lT_f - c_l T_c - \tau) + n(t), \tag{4}$$

where

$$w(t) = \sum_{k=0}^{N_{tap}-1} h_k \psi_r(t - kT_c). \tag{5}$$

Here P_r is the power received at a distance of one meter from the transmitter, $\psi_r(t)$ is the received UWB pulse of duration $T_w < T_c$ normalized to have unit energy, τ is the propagation delay, and $n(t)$ is an additive white gaussian noise (AWGN) process with zero mean and power spectral density σ^2.

III. Motivation and Problem Formulation

The timing information of the transmitted signal is essential for the performance of a receiver in a wireless communication system. In a multipath channel, the energy corresponding to the true signal phase is spread over several MPCs. Considering that we have no information regarding the channel state, EGC is a practical way of utilizing this energy to develop a more efficient acquisition scheme. The main difference between the acquisition problems in a multipath channel and a channel without multipath is that there are more than one hypothesized phases which can be considered a good estimate of the true signal phase. In a multipath environment, the receiver may lock onto a non-line-of-sight (non-LOS) path and still be able to perform adequately as long as it is able to collect enough energy. Fig. 1 shows two situations where a receiver performing EGC in a multipath environment is able to collect a considerable fraction of the energy of the received signal even though the estimated phase is not exactly equal to the true phase. For the purpose of illustration, we assume that the channel response has length $N_{tap} = 5$ and that the receiver is using an EGC window of length $G = 5$. The received signal is shown over a duration of one pulse repetition time $T_f = 7T_c$, where the TH sequence is providing an additional time shift of T_c. The shaded region correponds to the span of the EGC window. In the first situation, the hypothesized phase exceeds the true phase by T_c and in the second situation the hypothesized phase precedes the true phase by T_c. In both cases, the correlation of the received signal with the template signal will result in the collection of signal energy comparable to the energy collected in the case when the hypothesized phase equals the true phase. The difference between the two situations is that a receiver not performing EGC will not work for the situation shown in Fig. 1(b). It will work for the situation shown in Fig. 1(a) provided that the non-LOS MPC it locks onto is strong enough.

A typical acquisition system correlates the received waveform with a locally generated replica and compares the correlator output to a threshold to determine whether the hypothesized phase of the replica is correct. If the threshold is exceeded, the hypothesized phase becomes the estimate of the true phase. We define the *hit set* to be the set of estimated phases which result in successful demodulation. We assume that a particular bit will be correctly demodulated if the energy of the bit collected by the observation window is significantly greater than the energy of the adjacent bits collected by the observation window. Employing such an energy-based definition of a hit has the advantage of being independent of the particular modulation format used.

We assume that the normalized received monocycle waveform $\psi_r(t)$ and the TH sequence $\{c_l\}$ are known to the receiver. We propose to use an equal gain combiner of window size G. The receiver template signal $w_r(t)$ is given by

$$w_r(t) = \sum_{k=0}^{G-1} \psi_r(t - kT_c). \tag{6}$$

The reference TH signal can be obtained by combining the receiver template signal $w_r(t)$ and the known time hopping sequence as

$$s(t) = \sum_{l=0}^{MN_{th}-1} w_r(t - lT_f - c_l T_c - \hat{\tau}), \tag{7}$$

where M specifies the number of TH waveform periods in the dwell time and $\hat{\tau}$ is the hypothesized phase. To simplify the analysis, we assume that the true phase τ is an integer multiple of T_c. By the periodicity of the transmitted signal, we have $0 \leq \tau \leq (N_{th}N_h - 1)T_c$. The hypothesized phase $\hat{\tau}$ is also an integer multiple of T_c with the same range as τ. Then

$\Delta\tau = \hat{\tau} - \tau = \alpha T_f + \beta T_c$ where α and β are integers such that $-N_{th} + 1 \leq \alpha \leq N_{th} - 1$ and $0 \leq \beta \leq N_h - 1$. The correlator output is given by

$$\begin{aligned} y &= \int_{\hat{\tau}}^{\hat{\tau}+MN_{th}T_f} r(t)s(t)dt \\ &= \underbrace{M\sqrt{P_r}\sum_{k=0}^{N_{tap}-1} r_k(\Delta\tau)h_k}_{R(\Delta\tau;\mathbf{h})} + n_y, \end{aligned} \tag{8}$$

where $r_k(\Delta\tau)$ is the number of times the k^{th} MPC is collected by one period of the reference TH signal, $\mathbf{h}$ is an $N_{tap} \times 1$ vector containing the channel taps and n_y is the noise component of the correlator output. We note that n_y is a zero-mean Gaussian random variable with variance σ_y^2. If $S = \lceil \frac{N_h+G-2}{N_h} \rceil$ is the number of T_f time slots the EGC window can possibly reach and $S_m = \lceil \frac{N_{tap}-1}{N_h} \rceil$ is the number of T_f time slots the MPCs can spread across where $\lceil x \rceil$ is the smallest integer not less than x, we have

$$r_k(\Delta\tau) = \sum_{l=0}^{N_{th}-1} \sum_{i=-S_m}^{S} \sum_{j=0}^{G-1} \chi(c_l + j + \beta, c_{l+i+\alpha} + k + iN_h), \tag{9}$$

where $\chi(a, b) = 1$ if $a = b$, and 0 otherwise. The values of $r_k(\Delta\tau)$ and σ_y^2 depend on the particular pseudorandom TH sequence chosen. To simplify the analysis we assume that the TH sequence is random and that N_{th} is large. Under these assumptions the expected values of $r_k(\Delta\tau)$ and σ_y^2 are a reasonable approximation to the actual values. The expected values of $r_k(\Delta\tau)$ and σ_y^2 over the set of random TH sequences are calculated in the Appendix.

IV. Hit Set Definition

As suggested in the previouss discussion, a hypothesized phase belongs to the hit set if it results in succesful demodulation subsequent to acquisition. It seems reasonable to assume that a receiver equipped with the necessary hardware to perform EGC for acquisition will perform EGC for demodualtion as well. Thus we assume that a single EGC window size is used for both acquisition and demodulation. Let N_b consecutive UWB monocycles be modulated by one bit. To make a decision on a particular bit we correlate the received signal with a reference TH signal given by

$$s_b(t) = \sum_{l=0}^{N_b-1} w_r(t - lT_f - c_l T_c - \hat{\tau}). \tag{10}$$

The amount of a bit's energy collected by the correlator depends on the estimated phase $\hat{\tau}$. For successful demodulation of a bit, it is not sufficient if the energy of the bit collected by the correlator is significant. There is an additional restriction that this energy should be much greater than the energy of the adjacent bits collected by the correlator. Fig. 2 shows a simplified situation where a bit is incorrectly demodulated even though the correlator collects a significant amount of its energy. Suppose that the modulation scheme is bipolar amplitude modulation where a '1' is transmitted as a positive pulse and a '0' is transmitted as a negative pulse. For the purpose of illustration, we assume that a bit modulates a single monocycle, i.e. $N_b = 1$, the channel response has length $N_{tap} = 2$, the receiver is using an EGC window of length $G = 2$ and $N_h = 3$. The received signal corresponding to the bit sequence 10 is shown over a duration $2T_f$ where the TH sequence is providing additional time shifts of T_c and 0 for the first and second pulses respectively. the shaded region corresponds to the energy of the received signal collected by the correlator to make a decision on the first bit when the hypothesized phase exceeds the true phase by T_c. Even though the correlator collects the energy in the non-LOS MPC of the first bit it will fail to demodulate that bit correctly because it collects the LOS MPC of the second bit which is different from the first one. Thus we need to look at the difference of the collected energies corresponding to the desired bit and its adjacent bits in order to quantify successful demodulation.

For a given $\Delta\tau$, the difference of the correlator outputs corresponding to the desired bit and the adjacent bits is given by

$$R_b(\Delta\tau; \mathbf{h}) = \sqrt{P_r} \sum_{k=0}^{N_{tap}-1} r_k^b(\Delta\tau) h_k, \tag{11}$$

where

$$r_k^b(\Delta\tau) = \sum_{l=0}^{N_b-1} \sum_{i=-S_m}^{S} \sum_{j=0}^{G-1} \chi(c_l + j + \beta, c_{l+i+\alpha} + k + iN_h) \Delta(l + i + \alpha, 0) \Delta(N_b - 1, l + i + \alpha), \tag{12}$$

with $\Delta(a,b) = 1$ if $a \geq b$, and -1 otherwise. We note that $R_b(\Delta\tau; \mathbf{h}) > 0$ indicates that the energy collected from the desired bit is larger than that collected from the adjacent bits, and if this is the case, $R_b^2(\Delta\tau; \mathbf{h})$ is the effective bit energy which will determine system performance. Thus for a given true phase τ the hit set is defined as the following:

$$S_h = \{\hat{\tau} : \Pr[R_b(\Delta\tau; \mathbf{h}) > 0] > 1 - \lambda_1 \text{ and } \Pr[R_b^2(\Delta\tau; \mathbf{h}) < AR_b^2(0; \mathbf{h})] < \lambda_2\}, \tag{13}$$

where $A < 1$ is the fraction of the energy needed for successful demodulation of the bit given that the bit is correctly demodulated in the case of perfect synchronization and $0 < \lambda_1, \lambda_2 < 1$. The first condition in the definition guarantees that for the phases in the hit set, the energy of the desired bit collected by the correlator exceeds the energy of the adjacent bits collected by the correlator with high probability. The second condition guarantees that the effective bit energy for the phases in the hit set exceeds a fixed fraction of the effective bit energy in the case of perfect synchronization with high probability. The hit set typically consists of phases around the true phase as illustrated in Fig. 3(a) where the phases in the shaded region belong to the hit set. Owing to the outage formulation, it is also possible that the hit set consists of non-contiguous phases as shown in Fig. 3(b).

V. Probabilities of Detection and False Alarm

For a particular channel realization and fixed $\Delta\tau$, the correlator output y in (8) has a Gaussian distribution with pdf

$$p_Y(y) = \frac{1}{\sqrt{2\pi}\sigma_y} e^{-(y-R(\Delta\tau;\mathbf{h}))^2/2\sigma_y^2}. \tag{14}$$

The probabilities of false alarm and detection conditioned on the particular channel realization and given the decision threshold γ are given as

$$P_{FA}(\gamma, \Delta\tau) = \Pr[y > \gamma | \hat{\tau} \notin S_h] = Q\left(\frac{\gamma - R(\Delta\tau; \mathbf{h})}{\sigma_y}\right), \hat{\tau} \notin S_h. \tag{15}$$

$$P_D(\gamma, \Delta\tau) = \Pr[y > \gamma | \hat{\tau} \in S_h] = Q\left(\frac{\gamma - R(\Delta\tau; \mathbf{h})}{\sigma_y}\right), \hat{\tau} \in S_h. \tag{16}$$

The false alarm probability averaged over the channel realizations is given by

$$E_h[P_{FA}(\gamma,\Delta\tau)] \;=\; E_h\left[Q\left(\frac{\gamma - R(\Delta\tau;\mathbf{h})}{\sigma_y}\right)\right]. \tag{17}$$

The Gil-Pelaez lemma [17] gives an alternative form of the Q function as

$$Q(x) = \frac{1}{2} - \frac{1}{\pi}\int_0^\infty \frac{1}{t}e^{-t^2/2}\sin(tx)dt. \tag{18}$$

Substituting this form of the Q function we get

$$\begin{aligned}
E_h[P_{FA}(\gamma,\Delta\tau)] &= E_h\left[\frac{1}{2} - \frac{1}{\pi}\int_0^\infty \frac{1}{t}e^{-t^2/2}\sin t(\frac{\gamma - R(\Delta\tau;\mathbf{h})}{\sigma_y})dt\right] \\
&= \frac{1}{2} - \frac{1}{\pi}\int_0^\infty \frac{1}{t}e^{-t^2/2}E_h\left[\sin t(\frac{\gamma - R(\Delta\tau;\mathbf{h})}{\sigma_y})\right]dt \\
&= \frac{1}{2} - \frac{1}{\pi}\int_0^\infty \frac{1}{t}e^{-t^2/2}\mathrm{Im}\left\{E_h\left[e^{j(\frac{\gamma - R(\Delta\tau;\mathbf{h})}{\sigma_y})t}\right]\right\}dt \\
&= \frac{1}{2} - \frac{1}{\pi}\int_0^\infty \frac{1}{t}e^{-t^2/2}\mathrm{Im}\left\{e^{\frac{j\gamma t}{\sigma_y}}E_h\left[e^{\frac{-jR(\Delta\tau;\mathbf{h})t}{\sigma_y}}\right]\right\}dt \\
&= \frac{1}{2} - \frac{1}{\pi}\int_0^\infty \mathrm{Im}\left\{\frac{1}{t}e^{-t^2/2}e^{\frac{j\gamma t}{\sigma_y}}\phi_R\left(\frac{-t}{\sigma_y}\right)\right\}dt \\
&= \frac{1}{2} - \frac{1}{2\pi}\int_0^\infty \mathrm{Im}\left\{\frac{1}{t}e^{-t+\frac{j\gamma\sqrt{2t}}{\sigma_y}}\phi_R\left(\frac{-\sqrt{2t}}{\sigma_y}\right)\right\}dt,
\end{aligned} \tag{19}$$

where ϕ_R is the characteristic function of $R(\Delta\tau;\mathbf{h})$. From (8), $R(\Delta\tau;\mathbf{h})$ is a linear combination of independent random variables and hence its characteristic function is given by

$$\phi_R(\omega) = \prod_{k=0}^{N_{tap}-1}\phi_k(M\sqrt{P_r}r_k(\Delta\tau)\omega), \tag{20}$$

where $\phi_k(\cdot)$'s are the characteristic functions of the Nakagami-*m* distributed h_k's. Since finite limits of integration are more suitable for numerical integration we substitute $t = \tan\theta$ to get

$$E_h[P_{FA}(\gamma,\Delta\tau)] = \frac{1}{2} - \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \mathrm{Im}\left\{\frac{e^{-\tan\theta+\frac{j\gamma\sqrt{2\tan\theta}}{\sigma_y}}}{\sin 2\theta}\phi_R\left(\frac{-\sqrt{2\tan\theta}}{\sigma_y}\right)\right\}d\theta. \tag{21}$$

We get a similar expression for the average probability of detection.

VI. Mean Acquisition Time Analysis of Serial Search

We choose the decision threshold γ_d to be the minimum threshold such that the maximum of the average false alarm probability over $\hat{\tau} \notin S_h$ is not greater than a given constant δ.

$$\gamma_d = \underset{\gamma}{\operatorname{argmin}} \max_{\hat{\tau} \notin S_h} E_h[P_{FA}(\gamma, \Delta\tau)] \leq \delta. \tag{22}$$

We define a *hit* or acquisition event as the event when the decision threshold is exceeded for some $\hat{\tau} \in S_h$. We define a *miss* as the event when the decision threshold is not exceeded for all $\hat{\tau} \in S_h$. If the correlator outputs for different phase evaluations are assumed to be independent, then the average probability of a hit for a particular $\hat{\tau}$ is $E_h[P_D(\Delta\tau)]$ and the average probability of a miss is given by

$$P_M = \prod_{\hat{\tau} \in S_h} (1 - E_h[P_D(\gamma, \Delta\tau)]). \tag{23}$$

Although the average probability of a miss is a potential indicator of acquisition system performance, a more appropriate metric is the mean acquisition time. The mean acquisition time depends on the search strategy employed and to illustrate the effect of the choice of G, we assume that a serial search strategy is used. Owing to our definition, the hit set S_h consists of one or more clusters of hypothesized phases within the search space. The search space is the set $S_p = \{nT_c : n \in Z \text{ and } 0 \leq n \leq N_s - 1\}$ where $N_s = N_{th}N_h$. Let S_h consist of L disconnected clusters of phases within the search space where the size of the l^{th} cluster is H_l. Let H be the size of S_h where $H = \sum_{l=1}^{L} H_l$. Let the location of the l^{th} cluster within the search space be A_l where $1 \leq A_1 \leq A_2 \leq \ldots \leq A_L \leq N_s$. We will find it convenient to define B_l to be the position of the last phase of the l^{th} cluster where $H_l = B_l - A_l + 1$. Thus the l^{th} cluster consists of the phases $\{(A_l - 1)T_c, A_lT_c, \ldots, (B_l - 1)T_c\}$. The initial value of the hypothesized phase which corresponds to the starting point of the search is chosen at random from the set S_p.

We need to consider all possible sequences of events leading to a hit or acquisition event. The mean acquisition time can then be calculated as the average of the times taken for each of the acquisition events. An acquisition event is defined by a particular position n of the initial value of the hypothesized phase in S_p, the position (l, i) of the hypothesized phase in S_h where we have a hit where l is the index of the cluster and i is the position of the phase within that cluster, a particular number of misses j of S_h, and a particular number of false alarms k in hypothesized phases evaluated which do not belong to S_h. Let $T_{acq}(n)$ be the mean acquisition time conditioned on the event that the serial search starts at the n^{th} position in S_p i.e. the initial value of the hypothesized phase is $(n-1)T_c$. Then the mean acquisition time is

$$\bar{T}_{acq} = \frac{1}{N_s}\sum_{n=1}^{N_s} T_{acq}(n). \tag{24}$$

Without loss of generality, we can assume that $A_1 = 1$. First, suppose that the initial value of the hypothesized phase lies to the right of the last cluster i.e. $n \in \{B_L+1, B_L+2, \ldots, N_s\}$. The total acquisition time for a particular acquisition event defined by (n, j, k, l, i) is then

$$\begin{aligned} T(n,j,k,l,i) &= (N_s - n + 1)T + jN_sT + kT_{fa} + (A_l + i - 1)T \\ &= (N_s - n + jN_s + A_l + i)T + kT_{fa} \end{aligned} \tag{25}$$

where T is the dwell time for the evaluation of one hypothesized phase and T_{fa} is the time required to reject a hypothesized phase not in S_h when a false alarm occurs. The total number of hypothesized phases evaluated for this event is $N_s - n + jN_s + A_l + i$, the total number of evaluated phases which belong to S_h is $jH + \sum_{m=1}^{l-1} H_m + i$, and the total number of evaluated phases which do not belong to S_h is thus $K = N_s - n + j(N_s - H) + A_l - \sum_{m=1}^{l-1} H_m$. Let $P_d(l,i)$ denote the average probability of detection of the i^{th} phase in the l^{th} cluster of the hit set. The average probability of the serial search missing the l^{th} cluster is $P_M(l) = \prod_{i=1}^{H_l}[1 - P_d(l,i)]$. Let $P_M = \prod_{l=1}^{L} P_M(l)$ be the average probability of the serial search missing S_h. Then the probability of j misses of S_h followed by a hit at the phase in S_h which is at the i^{th} position of the l^{th} cluster is $P_M^j P_h(l,i) \prod_{c=1}^{l-1} P_M(c)$ where $P_h(l,i) = P_d(l,i)\prod_{r=1}^{i-1}[1 - P_d(l,r)]$. To simplify the analysis, we assume that the average probability of false alarm at the each of the phases not in S_h is $P_{fa} = \delta$. Then the average probability of the occurrence of k false alarms is given by $\binom{K}{k} P_{fa}^k (1 - P_{fa})^{K-k}$. To see the reasoning behind this assumption, recall that the decision threshold is chosen such that the maximum of the average false alarm probability over all hypothesized phases not in S_h is not greater than δ. Thus there may be phases not in S_h where the average probability of false alarm is less than δ. The average probability of the occurrence of k false alarms will involve a summation over $\binom{N_s-H}{k}$ events with distinct probabilities making the analysis intractable. The mean acquisition time obtained by making this assumption will be an upperbound for the actual mean acquisition time.

The mean acquisition time conditioned on the starting point of the serial search is given by

$$T_{acq}(n) = \sum_{l=1}^{L}\sum_{i=1}^{H_l}\sum_{j=0}^{\infty}\underbrace{\sum_{k=0}^{K}T(n,j,k,l,i)\binom{K}{k}P_{fa}^{k}(1-P_{fa})^{K-k}}_{T(n,j,l,i)}P_M^{j}P_h(l,i)\prod_{c=1}^{l-1}P_M(c). \quad (26)$$

Evaluating the innermost summation we get

$$\begin{aligned} T(n,j,l,i) &= \sum_{k=0}^{K}[kT_{fa}+(N_s-n+jN_s+A_l+i)T]\binom{K}{k}P_{fa}^{k}(1-P_{fa})^{K-k} \\ &= KP_{fa}T_{fa}+(N_s-n+jN_s+A_l+i)T \\ &= [N_s-n+j(N_s-H)+A_l-\sum_{m=1}^{l-1}H_m]P_{fa}T_{fa} \\ &\quad +(N_s-n+jN_s+A_l+i)T. \end{aligned} \quad (27)$$

Averaging over the number of misses j of the hit set we get

$$\begin{aligned} T(n,l,i) &= \sum_{j=0}^{\infty}T(n,j,l,i)P_M^{j} \\ &= \frac{(N_s-n+A_l)(T+P_{fa}T_{fa})+iT-P_{fa}T_{fa}\sum_{m=1}^{l-1}H_m}{1-P_M} \\ &\quad +\frac{N_sT+(N_s-H)P_{fa}T_{fa}}{(1-P_M)^2}P_M. \end{aligned} \quad (28)$$

Averaging over the position i within the l^{th} cluster we get

$$\begin{aligned} T(n,l) &= \sum_{i=1}^{H_l}T(n,l,i)P_h(l,i) \\ &= \frac{(N_s-n+A_l)(T+P_{fa}T_{fa})-P_{fa}T_{fa}\sum_{m=1}^{l-1}H_m}{1-P_M}(1-P_M(l)) \\ &\quad +\frac{\sum_{i=1}^{H_l}iTP_h(l,i)}{1-P_M}+\frac{N_sT+(N_s-H)P_{fa}T_{fa}}{(1-P_M)^2}P_M(1-P_M(l)), \end{aligned} \quad (29)$$

where we have used the identity $\sum_{i=1}^{H_l}P_h(l,i)=1-P_M(l)$. Finally, averaging over the cluster index l we get

$$
\begin{aligned}
T_{acq}(n) &= \sum_{l=1}^{L} T(n,l) \prod_{c=1}^{l-1} P_M(c) \\
&= (N_s - n)(T + P_{fa}T_{fa}) + \frac{N_s T + (N_s - H)P_{fa}T_{fa}}{(1-P_M)} P_M \\
&\quad + \frac{\sum_{l=1}^{L} [A_l(T + P_{fa}T_{fa}) - P_{fa}T_{fa} \sum_{m=1}^{l-1} H_m](1 - P_M(l)) \prod_{c=1}^{l-1} P_M(c)}{1 - P_M} \\
&\quad + \frac{\sum_{l=1}^{L} \sum_{i=1}^{H_l} iTP_h(l,i) \prod_{c=1}^{l-1} P_M(c)}{1 - P_M}, \qquad (30)
\end{aligned}
$$

where we have used the identity $\sum_{l=1}^{L}(1 - P_M(l)) \prod_{c=1}^{l-1} P_M(c) = 1 - P_M$.

Now suppose that the initial value of the hypothesized phase falls in the last cluster of S_h i.e. $n \in \{A_L, A_L + 1, \ldots, B_L\}$. Let m be the total number of phases evaluated for a particular acquisition event. We can partition the set of acquisition events into two sets, one containing those events for which $m \leq B_L - n + 1$ and the other containing those events for which $m > B_L - n + 1$. The mean acquisition time for events in the first set is just mT and for events in the second set it is $T_{acq}(B_L + 1) + (B_L - n + 1)T$ where $T_{acq}(B_L + 1)$ is obtained from (30). Averaging over the total number of phases evaluated we get

$$
\begin{aligned}
T_{acq}(n) &= \sum_{i=n}^{B_L} (i - n + 1)TP_d(L,i) \prod_{j=n}^{i-1} (1 - P_d(L,j)) \\
&\quad + (T_{acq}(B_L + 1) + (B_L - n + 1)T) \prod_{j=n}^{B_L} (1 - P_d(L,j)). \qquad (31)
\end{aligned}
$$

Now suppose the initial value of the hypothesized phase falls between the $(L-1)^{th}$ and L^{th} clusters i.e. $n \in \{B_{L-1} + 1, B_L, \ldots, A_L - 1\}$. The mean acquisition time in this case is the sum of the average time taken by the search process before it evaluates the first phase of the L^{th} cluster and $T_{acq}(A_L)$, the mean acquisition time conditioned on the event that the search process starts at the first phase of the L^{th} cluster which is obtained from (31).

$$
T_{acq}(n) = (A_L - n)(T + P_{fa}T_{fa}) + T_{acq}(A_L) \qquad (32)
$$

Proceeding in this recursive manner, we obtain the conditional mean acquisition times $T_{acq}(n)$ for all values of n. The mean acquisition time is obtained by substituting these values in (24).

VII. Results

In this section, we study the effect of the EGC window length G on the size of the hit set $|S_h|$, the average probability of a miss P_M and the mean acquisition time $\bar{T}_{acq}$ by choosing the following values for the system parameters. The TH sequence period $N_{th} = 1024$, $N_h = 16$, the length of the channel response $N_{tap} = 100$, and the number of monocycles modulated by one bit $N_b = 8$. We assume that $E_{tot} = -20.4$ dB which is its mean value when the transmitter-receiver (T-R) separation is 10 m [15]. We choose the power ratio $r = -4$ dB, decay constant $\epsilon = 16.1$ dB and fading figures $m_k = 3.5 - \frac{kT_c}{73}$, $0 \leq k \leq N_{tap} - 1$, which are their mean values given in [15].

Fig. 4 shows the variation of the size of the hit set as a function of the EGC window length for $\lambda_1 = \lambda_2 = 10^{-3}$ and the fraction of energy needed for successful demodulation $A = 0.5$ and 0.75. We note that as the fraction of energy needed for successful demodulation is increased, the size of the hit set decreases for a given EGC window length. This is due to the fact that there are fewer hypothesized phases which satisfy a more stringent energy requirement. There is an initial increase in the size of the hit set as G is increased because the EGC window is able to collect the required amount of energy for hypothesized phases moderately away from the true phase. Thus for small values of G, more hypothesized phases are included in the hit set as G is increased. For large values of G, we observe that the size of the hit set does not change significantly. This is due to the fact that for large G the energy of the adjacent bits collected by the EGC window is significant, reducing the effective bit energy $R_b^2(\Delta\tau; \mathbf{h})$ and preventing the inclusion of a significant number of hypothesized phases into the hit set.

Fig. 5 shows the effect of G on the probability of a miss P_M for $A = 0.5$ and 0.75. For each value of A, we plot P_M for the average energy to noise ratio $\frac{P_r E_{tot}}{\sigma^2} = -10$ dB and -20 dB. There is an substantial decrease in P_M for moderate values of G but the decrease is insignificant when G is large.

To study the effect of G on the mean acquisition time $\bar{T}_{acq}$, we assume that the dwell time is equal to one period of the TH sequence i.e. $M = 1$ and $T = N_{th}N_hT_c$. The false alarm penalty time T_{fa} is assumed to be $5T$. Fig. 6 shows the mean acquisition time in seconds as a function of G. For all the values of A and $\frac{P_r E_{tot}}{\sigma^2}$ considered, the mean acquisition time increases initially and then decreases significantly. For large G, the decrease in the mean acquisition time is marginal. The minimum mean acquisition time is seen to be of the order of a second which is too high from a practical system viewpoint. This is due to the large search space and the fact that the serial search has to evaluate a considerable number of phases on the average before it encounters the hit set. This issue can be alleviated by a parallel search strategy. We also observe that the mean acquisition time does not depend on the average energy to noise ratio for large values of G. This is due to the fact that a large EGC window collects a significant amount of energy even when the hypothesized phase is not in the hit set. This energy is comparable to the energy collected for phases in the hit set. Thus a threshold which constrains the maximum average probability of false alarm to be less than δ also constrains the average probability of detection to approximately the same value irrespective of the magnitude of the energy available in the multipaths.

VIII. Conclusions

We have analyzed an acquisition sytem for UWB signals which performs EGC to utilize the energy in the multipaths. By considering system performance subsequent to acquisition, the set of phases which can be considered a *hit* was obtained. With mean acquisition time as the metric for system performance, we observe that a significant gain can be obtained by a system performing EGC. The far from practical values of the mean acquisition time obtained motivate the need for a parallel search strategy and the development of acquisition schemes capable of reducing the search space.

APPENDIX

A. Average number of MPCs collected

The calculation of the expected value of $r_k(\Delta\tau)$ is made easy by the observation that it is a sum of Bernoulli distributed random variables.

$$r_k(\alpha T_{th} + \beta T_c) = \sum_{l=0}^{N_{th}-1} \sum_{i=-S_m}^{S} \underbrace{\sum_{j=0}^{G-1} \chi(c_l + j + \beta, c_{l+i+\alpha} + k + iN_h)}_{\text{Bernoulli random variable}} \tag{33}$$

Hence the expected value is just the sum of the probabilities of the events of each Bernoulli random variable taking the value 1.

$$\begin{aligned}
&\Pr[\sum_{j=0}^{G-1} \chi(c_l + j + \beta, c_{l+i+\alpha} + k + iN_h) = 1] \\
&= \Pr[\bigcup_{j=0}^{G-1} (c_{l+i+\alpha} + k + iN_h = c_l + j + \beta)] \\
&= \sum_{j=0}^{G-1} \Pr[c_{l+i+\alpha} + k + iN_h = c_l + j + \beta] \\
&= \sum_{j=0}^{G-1} \sum_{m} \Pr[c_{l+i+\alpha} + k + iN_h = m \mid c_l + j + \beta = m] \Pr[c_l + j + \beta = m] \\
&= \sum_{j=0}^{G-1} \sum_{m=j+\beta}^{j+\beta+N_h-1} \Pr[c_{l+i+\alpha} + k + iN_h = m \mid c_l + j + \beta = m] \Pr[c_l + j + \beta = m] \\
&= \sum_{j=0}^{G-1} \sum_{m=j+\beta}^{j+\beta+N_h-1} \frac{1}{N_h} \Pr[c_{l+i+\alpha} + k + iN_h = m \mid c_l + j + \beta = m] \\
&= \sum_{j=0}^{G-1} \sum_{m=j+\beta}^{j+\beta+N_h-1} \frac{1}{N_h} \Pr[c_{l+i+\alpha} + k + iN_h = m] \\
&= \sum_{j=0}^{G-1} \sum_{m=\max\{j+\beta, k+iN_h\}}^{\min\{j+\beta+N_h-1, k+iN_h+N_h-1\}} \frac{1}{N_h^2} \\
&= \sum_{j=0}^{G-1} \sum_{m=\max\{j+\beta, k+iN_h\}}^{\min\{j+\beta, k+iN_h\}+N_h-1} \frac{1}{N_h^2}
\end{aligned} \tag{34}$$

The sixth equality in the above calculation is due to the random sequence assumption which does not hold if $i + \alpha = 0 (mod\, N_{th})$. If there is an $i_1 \in \{-S_m, -S_m + 1, \ldots, S - 1, S\}$ such that $i_1 + \alpha = 0 \ (mod \ N_{th})$, we have

$$\begin{aligned} & \Pr[\sum_{j=0}^{G-1} \chi(c_l + j + \beta, c_{l+i_1+\alpha} + k + i_1 N_h) = 1] \\ &= \ U(\beta + G - 1, k + i_1 N_h) U(k + i_1 N_h, \beta), \end{aligned} \tag{35}$$

where $U(a,b) = 1$ if $a \geq b$ and 0 otherwise. In general, we have

$$\begin{aligned} r_k(\alpha T_{th} + \beta T_c) \ &= \ N_{th} U(\beta + G - 1, k + i_1 N_h) U(k + i_1 N_h, \beta) \\ &+ \sum_{l=0}^{N_{th}-1} \sum_{\substack{i=-S_m \\ i \neq i_1}}^{S} \underbrace{\sum_{j=0}^{G-1} \chi(c_l + j + \beta, c_{l+i+\alpha} + k + i N_h)}_{\text{Bernoulli random variable}} . \end{aligned} \tag{36}$$

The expected value of the second term on the right hand side is calculated as before.

B. Noise Variance Calculation

We assume the TH sequence to be a random sequence to calculate the average noise variance $E[\sigma_y^2]$ of the noise collected by the correlator.

$$\begin{aligned} \frac{\sigma_y^2}{\sigma^2} \ &= \ M N_{th} G + 3M \sum_{l=0}^{N_{th}-1} (\text{Overlap of } l^{th} \text{ and } (l+1)^{th} \text{ windows}) \\ &+5M \sum_{l=0}^{N_{th}-1} (\text{Overlap of } l^{th}, (l+1)^{th} \text{ and } (l+2)^{th} \text{ windows}) \\ &+7M \sum_{l=0}^{N_{th}-1} (\text{Overlap of } l^{th}, (l+1)^{th}, (l+3)^{th} \text{ and } (l+4)^{th} \text{ windows }) \\ &+(2k-1)M \sum_{l=0}^{N_{th}-1} (\text{Overlap of } l^{th}, (l+1)^{th}, \ldots, (l+k)^{th} \text{ windows}), \end{aligned} \tag{37}$$

where $k = \lceil \frac{G-1}{N_h} \rceil$. Overlap between two consecutive EGC windows is given by

$$\sum_{j=0}^{G-1} U(c_l + G - 1, c_{l+1} + N_h + j). \tag{38}$$

Similarly, overlap between three consecutive EGC windows is given by

$$\sum_{j=0}^{G-1} U(c_l + G - 1, c_{l+2} + 2N_h + j) U(c_{l+1} + N_h + G - 1, c_{l+2} + 2N_h + j). \tag{39}$$

Noting the fact that the second term in the above product is nonzero whenever the first term is nonzero the equation for the overlap reduces to

$$\sum_{j=0}^{G-1} U(c_l + G - 1, c_{l+2} + 2N_h + j). \tag{40}$$

So in general the overlap between i consecutive windows is given by

$$\sum_{j=0}^{G-1} U(c_l + G - 1, c_{l+i-1} + (i-1)N_h + j). \tag{41}$$

The expected value of the overlap between i consecutive windows is then given by

$$\sum_{k=1}^{\infty} k\text{Pr}[\text{Overlap of } i \text{ consecutive windows} = k], \tag{42}$$

where

$$\begin{aligned}
&\text{Pr}[\text{Overlap of } i \text{ consecutive windows} = k] \\
&= \text{Pr}[\sum_{j=0}^{G-1} U(c_l + G - 1, c_{l+i-1} + (i-1)N_h + j) = k] \\
&= \text{Pr}[c_l + G - 1 = c_{l+i-1} + (i-1)N_h + k - 1] \\
&= \text{Pr}[c_l + G = c_{l+i-1} + (i-1)N_h + k] \\
&= \sum_{j=0}^{N_h-1} \text{Pr}[c_{l+i-1} + (i-1)N_h + k = j + G \mid c_l = j]\text{Pr}[c_l = j] \\
&= \sum_{j=0}^{N_h-1} \frac{1}{N_h}\text{Pr}[c_{l+i-1} + (i-1)N_h + k = j + G \mid c_l = j] \\
&= \sum_{j=0}^{N_h-1} \frac{1}{N_h}\text{Pr}[c_{l+i-1} + (i-1)N_h + k = j + G] \\
&= \sum_{j=\max\{0, G-(i-1)N_h-k\}}^{\min\{N_h-1, G-(i-1)N_h-k+N_h-1\}} \frac{1}{N_h}\text{Pr}[c_{l+i-1} = j] \\
&= \sum_{j=\max\{0, G-(i-1)N_h-k\}}^{\min\{0, G-(i-1)N_h-k\}+N_h-1} \frac{1}{N_h^2}
\end{aligned} \tag{43}$$

The expected overlaps can be substituted in (37) to get the average noise variance.

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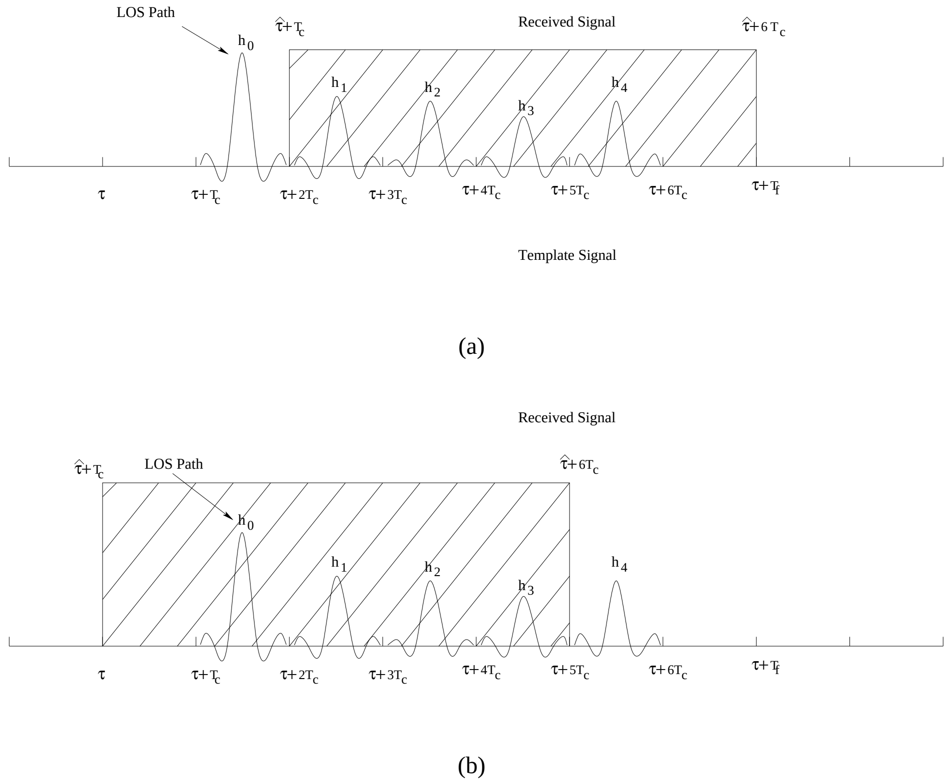

Fig. 1. Illustration of situations where the receiver may perform adequately even when it locks onto a non-line-of-sight path

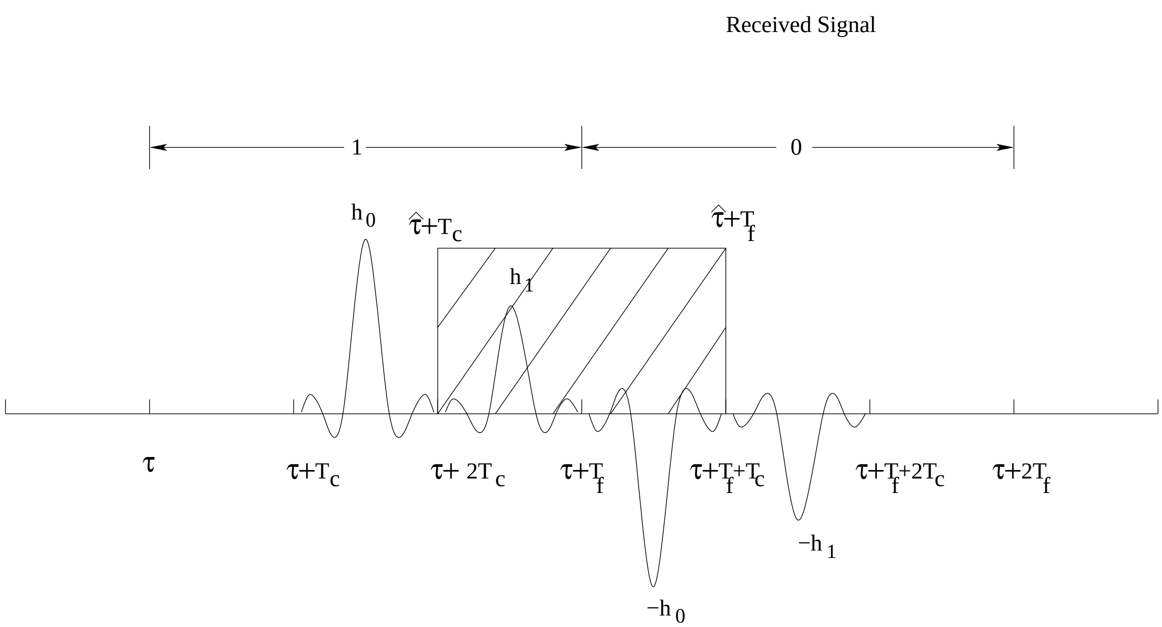

Fig. 2. Illustration of a situation where a bit is incorrectly demodulated due to the effect of the adjacent bit

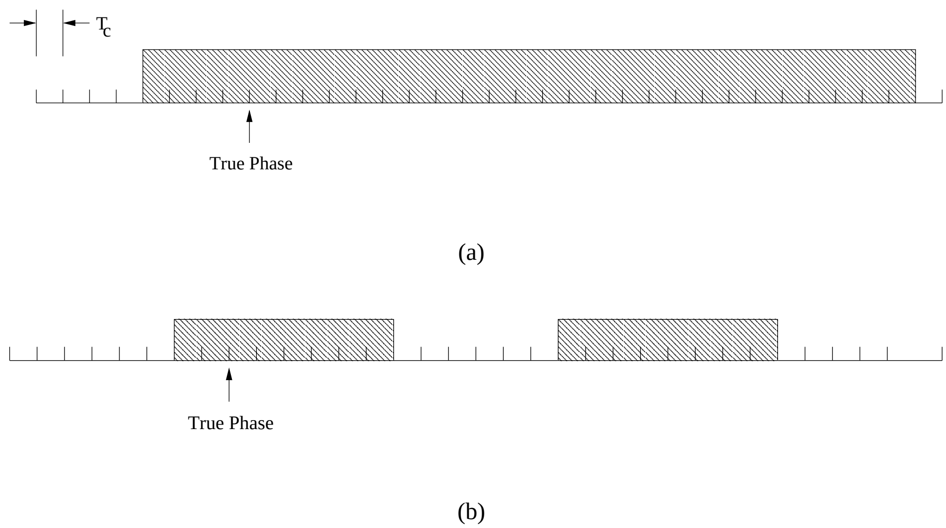

Fig. 3. Illustration of typical hit sets

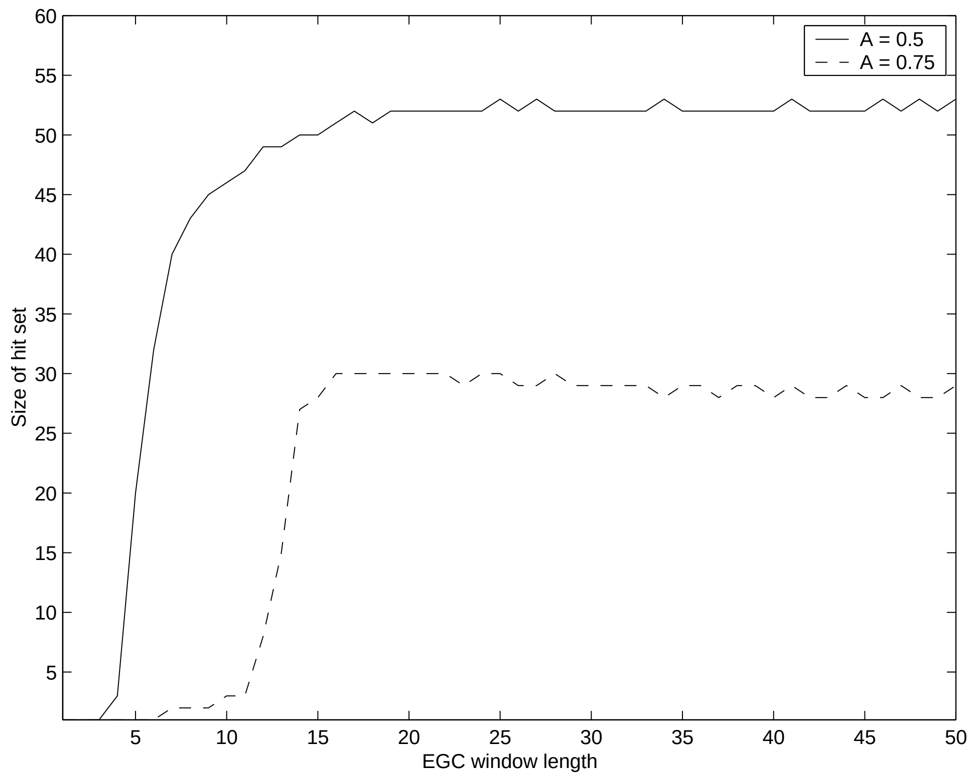

Fig. 4. Hit set size versus EGC window length for two values of A.

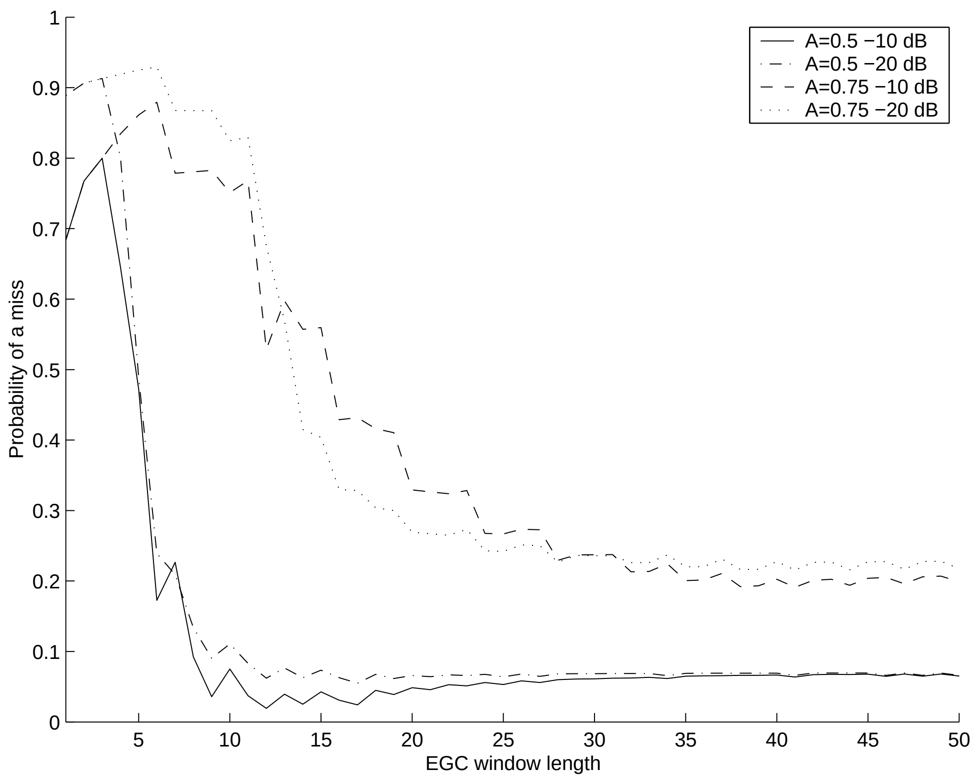

Fig. 5. Effect of EGC window length on the probability of a miss

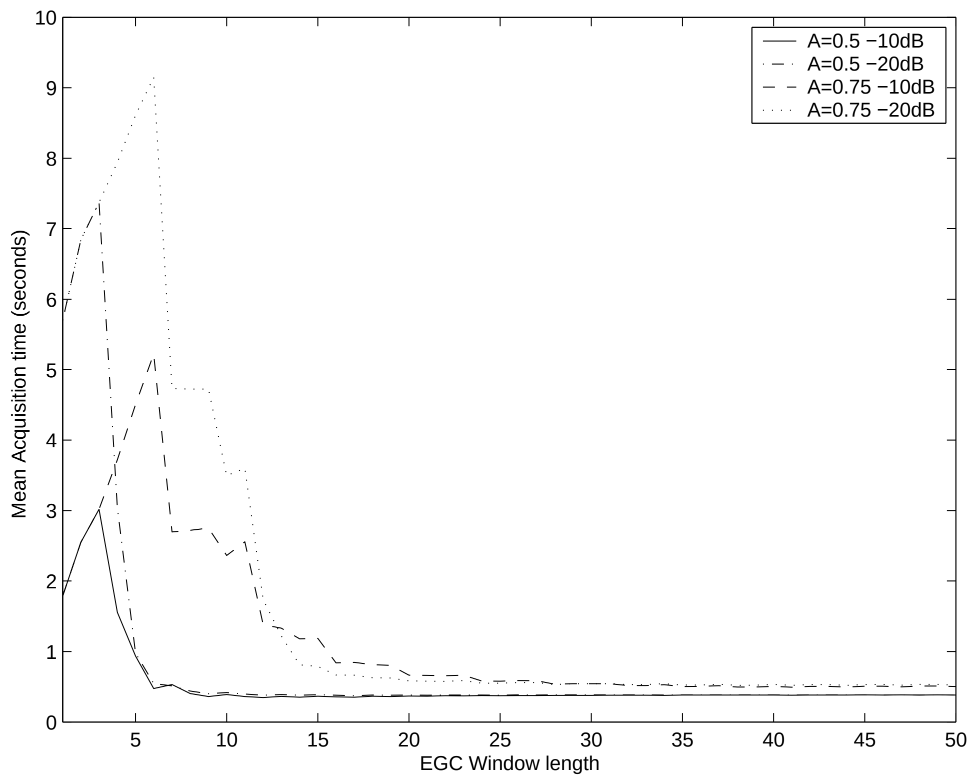

Fig. 6. Effect of EGC window length on the mean acquisition time