New Equalization Approach for OFDM over Dispersive and Rapidly Time Varying Channel

Jean-Paul M.G. LINNARTZ and Alexei GOROKHOV
Philips Research, Nat.Lab. WY,
Holstlaan 4, 5656 AA Eindhoven, The Netherlands
linnartz@ieee.org alexei.gorokhov@philips.com
http://www.eecs.berkeley.edu/~linnartz http://wireless.per.nl

Abstract - The paper proposes and analyses a new receiver structure to mitigate the effect of Doppler on the reception of OFDM signals. A Discrete-Frequency channel representation is developed for the link between the input of the transmit I-FFT and the output of the receive FFT. It is based on a Taylor expansion of the time variations of the received subcarrier amplitudes. The model realistically addresses the correlation of fading at neighboring subcarriers. We study a new type of receiver which estimates not only amplitudes but also derivatives of subcarriers amplitudes. An adaptive MMSE filter is proposed to cancel the Intercarrier Interference (ICI) resulting from Doppler. This results in a substantial improvement of the link performance.

Introduction

Orthogonal Frequency Division Multiplexing (OFDM) is a modulation method designed in the 1970's [1-6] in which multiple user symbols are transmitted in parallel using different subcarriers. Although these subcarriers have overlapping (sinc-shaped) spectra, the signal waveforms are orthogonal. Compared to other modulation methods, OFDM uses symbols with a relatively long time duration, but a narrow bandwidth. Mostly, OFDM systems are designed such that each subcarrier is small enough in bandwidth to experience frequency-flat fading. This also ensures that the subcarriers remain orthogonal when received over a (moderately) frequency selective but time-invariant channel. If the OFDM signal is received over such a channel, each subcarrier experiences a different attenuation, but no dispersion. These properties avoid the need for a tapped delay line equalizer. This has been a prime motivation to use OFDM in several standards, such as Digital Audio Broadcasting (DAB), e.g. [5], the Digital Terrestrial Television Broadcast (DTTB), and more recently the wireless local area network standard HIPERLAN II.

Particularly in the DAB and DTTB applications, mobile reception under disadvantageous channel conditions is foreseen, with both frequency and time dispersion. Mobile reception, i.e., reception over a Doppler channel is recognised as being one of the main problems associated with OFDM systems. Time variations are known to corrupt the orthogonality of the OFDM subcarrier waveforms [2,15]. In such case, Inter-Carrier Interference (ICI) also called "FFT leakage", occurs because signal components from one subcarrier spill into other, mostly to neighboring subcarriers. In this paper, we model mobile propagation with Doppler to address the effect of user mobility. While some analyses of the effect of Doppler appeared in open literature (e.g. [7,8,13,16]), countermeasures are relatively unknown or impractical. We believe that our approach allows substantial improvements of the link performance at limited receiver complexity, without requiring any modification to the transmit standard.

OFDM is often interpreted as the time-frequency dual of QAM. In OFDM, symbols are transmitted after a performing a (Inverse) Discrete Fourier Transform (DFT), in particular a Fast Fourier Transform (FFT), thus each symbol is located within its own frequency bin, but shares in a common time interval with other symbols. A Doppler (frequency) spread in OFDM can be interpreted as the dual of a (time) delay spread in high-rate QAM. In QAM, an equalizer can repair the delay spreads. The duality principle suggests that in OFDM, an FFT followed by a parallel-to-serial converter and a tapped line filter can be exploited to improve performance. The basic idea to repair InterCarrier Interference by adaptively combining subcarrier signals is intuitively appealing and can work if the delay spread is negligibly small. in such case all subcarriers experience the same amplitude and phase shifts, thus the ICI arrives with same crosstalk coefficients. However, for channels with both a delay and a Doppler spread, a practical implementation is more complicated, typically require a matrix multiplication. This matrix has its dominant contributions along the main diagonal, but is not of a Toeplitz structure that would allow a delay line filter structure. Moreover, extensive channel estimation poses problems in practice. So the prime challenge is to find a realistic channel representation which allows a computationally attractive implementation. We found this in the form of time-derivatives of amplitudes.

The outline of the paper is as follows: Section 2 revisits the model of a mobile radio channel with a large number of frequency and time-shifted scattered waves. In particular we will introduce a Tayler expansion of the trajectory of the complex amplitude. Section 3 uses the channel model to formulate a discrete-frequency representation of the system between the input of the I-FFT at the transmitter and the output of the FFT at the receiver. Section 4 proposes a receiver based on adaptive equalization of the ICI channel. Simulations are in section 5. Section 6 concludes the paper.

2. Channel Model

This section develops a model for a channel representation starting from the classic multipath description, using as a collection of I_w reflected waves. Each wave has its particular Doppler frequency offset ω_i, path delay T_i and amplitude D_i. The frequency offset lies within the Doppler spread $-2\pi f_\Delta < \omega_i < 2\pi f_\Delta$, with $f_\Delta = v_s f_c/c$. Here v_s is the velocity of the mobile antenna, c is the speed of light and the carrier frequency is $\omega_c = 2\pi f_c$. More precisely, the Doppler shift of the i-th wave is $\omega_i = (2\pi v_s/c)\ f_c \cos(\theta_i)$ with θ_i the angle of arrival. Let f_s denote the spacing between the adjacent subcarriers and $\omega_s = 2\pi f_s$. The received signal equals

$$r(t) = \sum_{n=0}^{N-1} \sum_{i=0}^{I_w-1} a_n D_i \exp\{j(w_c + nw_s)(t - T_i) + jw_i t\} + n(t) \quad (1)$$

here $n(t)$ is additive white Gaussian noise. We will denote the vector of modulation symbols as $\boldsymbol{A} = [a_0, a_1, ...a_{N-1}]^T$. The transmit energy per subcarrier is $E_N = \mathrm{E}\,|a_n|^2$. Not all reflected waves are individually 'resolvable', that is, a receiver sampling in a time-window of finite duration will only see the collective effect of multiple reflected waves within a certain time-frequency window.

The narrowband mobile channel model can be compacted into a complex received amplitude that is time varying. We rewrite (1) as

$$r(t) = \sum_{n=0}^{N-1} a_n V_n(t) \exp\{j(w_c + nw_s)t\} + n(t) \quad (2)$$

where the time-varying channel amplitude $V_n(t)$ at the n-th subcarrier is

$$V_n(t) = \sum_{i=0}^{I_w-1} D_i \exp\{-j(w_c + nw_s)T_i + jw_i t\} \quad (3)$$

A Tayler expansion is $V_n(t) = v_n^{(0)}(t_0) + v_n^{(1)}(t_0)\ (t - t_0) + v_n^{(2)}(t_0)\ (t - t_0)^2 / 2 + ..$. Here $v_n^{(q)}(t_0)$ denotes the q-th derivative of the amplitude with respect to time, at instant $t = t_0$. If the Doppler spread is much smaller than the frequency resolution of the FFT grid, we may restrict our analysis to zero and first order effects. Omitting the argument (t_0) in the notation of the derivative $v_n^{(q)}(t_0)$, we find

$$v_n^{(q)} = \sum_{i=0}^{I_w-1} \left(jw_i\right)^q D_i \exp\{-j(w_c + nw_s)T_i + jw_i t_0\} \quad (4)$$

Hence (5)

$$r(t) = \sum_{n=0}^{N-1} a_n \left(\sum_{q=0} \frac{v_n^{(q)}}{q!} (t - t_0)^q \right) \exp\{ j(w_c + nw_s)t\} + n(t)$$

In a Rayleigh channel, $v_n^{(q)}$ is zero-mean complex Gaussian for any n and q. To characterize the channel, we are interested in the covariance of variables $v_n^{(p)}$ and $v_m^{(q)}$, viz., $\mathrm{E}v_n^{(p)}v_m^{*(q)}$, where * denotes the complex conjugate. Note that in our case we may freely interchange the sequence of performing a complex conjugation and taking derivatives. so (6)

$$\mathrm{E}\, v_n^{(p)} v_m^{*(q)} = \mathrm{E} \sum_{i=0}^{I_w-1} \sum_{k=0}^{I_w-1} j^p w_i^p (-1)^q j^q w_k^q D_i D_k^* \bullet$$
$$\exp\left(-j(w_c + nw_s)T_i + jw_i t_0 + j(w_c + mw_s)T_k - jw_k t_0\right)$$

In the expectation $\mathrm{E}D_iD_k^*$, only contributions with $i = k$ remain. For the distribution of D_i, T_i and the Doppler shift ω_i, we insert the commonly used scatter function [9-11],

$$\lim_{I_w \to \infty} \mathrm{E} \sum_{i \in A_{\tau q}} D_i D_i^* = \frac{1}{2\pi T_{rms}} \exp\left(-\frac{t}{T_{rms}}\right) dt\, dJ \quad (7)$$

where

$$A_{\tau J} = \{i : t < T_i < t + dt\} \cap \{i : 2\pi f_\Delta \cos\theta < \omega_i < 2\pi f_\Delta \cos(\theta + d\theta)\} \quad (8)$$

This corresponds to an exponential delay spread and a uniform angle of arrival, with a local-mean received power of unity. In this paper, we normalize all received powers (including the AWGN noise) to this level. We define the frequency offset, expressed in units of subcarrier spacing, as $\Delta = n - m$. The covariance becomes

$$\mathrm{E}\, v_n^{(p)} v_m^{*(q)} = \frac{w_c^{p+q} v_s^{p+q}}{c^{p+q}} \frac{(-1)^q j^{p+q}}{2\pi T_{rms}} \int_0^{2\pi} \cos^{p+q}\theta\, d\theta \int_0^\infty \exp\left(-\left(\frac{1}{T_{rms}} + j\Delta w_s\right) t\right) dt \quad (9)$$

For $p + q$ even, this leads to

$$\mathrm{E}\, v_n^{(p)} v_m^{*(q)} = \left(2\pi f_\Delta\right)^{p+q} \frac{(p+q-1)!!}{(p+q)!!} \frac{(-1)^q j^{p+q}}{1 + j\Delta T_{rms} w_s} \quad (10)$$

and $\mathrm{E}v_n^{(p)}v_m^{*(q)} = 0$ for $p + q$ is odd. These equations define the covariance matrix of subcarrier amplitudes and derivatives, thereby allowing system modeling and simulation in discrete frequency domain, i.e., between the input of the transmit I-FFT and output of the receive FFT. Previously reported results e.g. in [9] confirm special cases of (10).

3 Subcarrier System Model

In OFDM, a frame of N symbols is detected by taking N samples and performing an FFT. Figure 1 depicts the channel and receiver in the discrete frequency domain. thus the FFT is not drawn explicitly. In a conventional system, $\boldsymbol{W}$ represents the equalizer, or automatic gain control per subcarrier, to compensate for fading on the subcarriers.

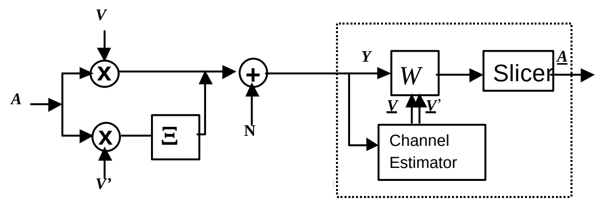


Figure 1: Discrete-Frequency representation for the Doppler multipath channel and the OFDM receiver

Detection of the signal at subcarrier m occurs by correlation with a complex sinusoid having the frequency of the m-th subcarrier, viz., $\exp\{-j\omega_c t - j(m-\Delta_f)\omega_s t\}$ during the symbol duration NT. Here Δ_f is the frequency offset normalized to the subcarrier spacing ω_s. In the sampling process, the receiver makes a timing error Δ_t, where Δ_t is the time offset normalized to a sample interval $T = (N f_s)^{-1}$. That is, it samples at $t = \Delta_t$, $\Delta_t + T$, $\Delta_t + 2T$, ... We assume that cyclic prefixes avoid any interframe interference. We use $\boldsymbol{Y}$ as the vector of length N to denote the output of the FFT in the receiver. The m-th output of the FFT is found as (11)

$$y_m = \frac{1}{N}\sum_{k=0}^{N-1} r(kT+\Delta_t)\exp\{-j(\omega_c + \omega_s(m+\Delta_f))(kT+\Delta_t)\} + n_m$$

Here, $m = 0, 1, ..., N - 1$ and $\boldsymbol{Y} = [y_0, y_1, ..., y_{N-1}]^T$. We observe that $\omega_s T = 2\pi/N$. We introduce the OFDM system parameter $\xi_\Delta^{(q)}$, defined as (12)

$$\xi_\Delta^{(q)} = \frac{1}{N}\sum_{k=0}^{N-1} k^q \exp\left\{\frac{2\pi jk\Delta}{N}\right\},$$

to describe the signal transfer over the q-th derivative of the amplitude at subcarier n to the $(n+\Delta)$-th receive subcarrier. This allows us to rewrite y_m as follows: (13)

$$y_m = \frac{1}{N}\sum_{n}^{N-1} a_n e^{j2\pi\left(f_c - \frac{\Delta_f}{N} + nf_s\right)\Delta_t} \sum_{q=0}^{\infty} \frac{v_n^{(q)}\xi_{n-m-\Delta_f}^{(q)} T^q}{q!} + n_m$$

In particular, we will consider the first two terms of the expansion, and we denote $\chi_\Delta = \xi_\Delta^{(0)}$ and $\zeta_\Delta = \xi_\Delta^{(1)}$. So,

$$c_\Delta = \frac{1}{N}\sum_{k=0}^{N-1}\exp\left\{\frac{2\pi jk\Delta}{N}\right\} = \frac{1}{N}\;\frac{1-\exp\{j2\pi\Delta\}}{1-\exp\{j2\pi\Delta/N\}} \quad (14)$$

For integer Δ, χ_Δ reduces to δ_Δ, which is just a confirmation that subcarriers (with a nonfading amplitude) are orthogonal. Note further that

$$\zeta_\Delta = -\frac{jN}{2\pi}\frac{c_\Delta}{d\Delta} \quad (15)$$

for integer and non-zero Δ, we see that

$$\zeta_\Delta = \begin{cases} (N-1)/2, & \Delta = 0 \bmod N \\ -(1-\exp\{j2\pi\Delta/N\})^{-1}, & \Delta \neq 0 \bmod N \end{cases} \quad (16)$$

Roughly speaking $\zeta_\Delta / \zeta_0 \approx (j\pi\Delta)^{-1}$, with Δ= 1, 2, 3, ..., so the ICI reduces slowly with increasing subcarrier separation. Relatively many subcarriers make a significant contribution to the ICI. We define vector $\boldsymbol{V} = [v_0, v_1, .., v_{N-1}]$ for the subcarrier amplitudes and $\boldsymbol{V'} = [v_0^{(1)}, v_1^{(1)}, .., v_{N-1}^{(1)}]^T$ for the derivatives. For an ideally synchronizing receiver, i.e., with $\Delta_t = 0$ and $\Delta_f = 0$,, the received signal $\boldsymbol{Y}$ can compactly be written in Discrete Frequency domain as,

$$\boldsymbol{Y} = (\,\mathrm{DIAG}(\boldsymbol{V}) + T\,\Xi\,\mathrm{DIAG}(\boldsymbol{V'})\,)\,\boldsymbol{A} + \boldsymbol{N} \quad (17)$$

with DIAG(X) the diagonal matrix composed of the elements of vector X, and

$$\Xi = \begin{bmatrix} \zeta_0 & \zeta_1 & .. & \zeta_{N-1} \\ \zeta_{-1} & \zeta_0 & .. & \zeta_{N-2} \\ .. & .. & .. & .. \\ \zeta_{-N+1} & \zeta_{-N+2} & .. & \zeta_0 \end{bmatrix} \quad (18)$$

Not all terms in Ξ address ICI: In (17) the diagonal terms ζ_0 carry signal components from the n subcarrier to the n-th subcarrier, so in fact the receiver sees a signal amplitude of $(\chi_0 v_n + \zeta_0 T v_n^{(1)})a_n$. To address this in the following calculations we define $\Xi^* = \Xi - \zeta_0 \boldsymbol{I}_N$. The wanted signal component in a conventional receiver equals $\boldsymbol{V}+\zeta_0\boldsymbol{V'}T$, which in practice closely approximates $\boldsymbol{V}$, except in deep fades.

3.1 Average SINR

Just to verify our results this far, we use our model to estimate the average Signal-to-ICI-plus-Noise (SINR) for OFDM with Doppler and compare it with results from previously proposed models for symtems with many subcarriers (large N). We compute the signal-to-ICI-plus-noise from the variance of the ICI,

$$\sigma_{ICI}^2 = \sum_{\substack{m=0 \\ m\neq n}}^{N-1} \zeta_{n-m}\zeta_{n-m}^* T^2 E_N \,\mathrm{E}\, v_m^{(1)} v_m^{(1)*} \approx E_N \frac{f_\Delta^2}{f_s^2}\sum_{i=1}^{\infty}\frac{1}{i^2} \quad (19)$$

The expected signal-to-noise ratio becomes

$$\gamma_{av} = \frac{E_N}{\frac{\pi^2 f_\Delta^2}{6 f_s^2} E_N + N_0} \quad (21)$$

with N_0 the spectral power density of the noise. This agrees with results in [7,8,13,16], where a continuous U-shaped Doppler spectrum was considered. In [13], a result equivalent to (21) was computed by simplifying the sinc-shaped frequency-domain window by a linear approximation in each sampling point. An interactive spreadsheet to evaluate ICI is available on the web [15].

4 Receiver Design

The OFDM receiver sees the signal $\boldsymbol{Y} = \boldsymbol{Q}\,\boldsymbol{A} + \boldsymbol{N}$, with matrix $\boldsymbol{Q} = \mathrm{DIAG}(\boldsymbol{V}) + T\,\Xi\,\mathrm{DIAG}(\boldsymbol{V}')$. Our OFDM receiver is extended such that it can not only reliably estimate amplitudes (as conventional receivers do), but also complex valued derivatives (which is not common for normal OFDM receivers), with $\underline{\boldsymbol{V}}$ and $\underline{\boldsymbol{V}}'$ the estimate of amplitudes and derivatives, respectively. Then the data can be recovered as follows:

- Create the matrix $\underline{\boldsymbol{Q}} = \mathrm{DIAG}(\underline{\boldsymbol{V}}) + T\,\Xi\,\mathrm{DIAG}(\underline{\boldsymbol{V}}')$.
- Compute an MMSE filter $\boldsymbol{W}$ according to
  $\boldsymbol{W} = \underline{\boldsymbol{Q}}^{\mathrm{H}}\,[\,\underline{\boldsymbol{Q}}\,\underline{\boldsymbol{Q}}^{\mathrm{H}} + N_0\, I_N\,]^{-1}$.

For a receiver that perfectly estimates the channel, the covariance matrix of the residual ICI plus noise, normalized to the signal power, becomes

$$C = \left[I_N + \frac{E_n}{N_0} \boldsymbol{Q}^H \boldsymbol{Q} \right]^{-1} \qquad (22)$$

The vector of the SINR at the N subcarriers is

$$\mathrm{Dg}(\boldsymbol{Q})\ \mathrm{Dg}(\boldsymbol{Q})^{\mathrm{H}} E_N \,./\, \mathrm{diag}(\boldsymbol{C}), \qquad (23)$$

where ./ is a subcarrier-by-subcarrier division and Dg(X) is a matrix identical to X on the main diagonal and zero elsewhere, i.e., Dg(X) = DIAG(diag(X)). A conventional link with known channel characteristics would have a noise plus ICI contribution described by covariance matrix:

$$E_N\, T^2 \Xi^* \boldsymbol{V}' \boldsymbol{V}'^{\mathrm{H}} \Xi^{*\mathrm{H}} + N_0\, I_N, \qquad (24)$$

The vector of SINR values is found as

$$\mathrm{Dg}(\boldsymbol{Q})\ \mathrm{Dg}(\boldsymbol{Q})^{\mathrm{H}} E_N \,./\mathrm{diag}(T^2 \Xi^* \boldsymbol{V}' \boldsymbol{V}'^{\mathrm{H}} \Xi^{*\mathrm{H}} E_N + N_0 \boldsymbol{I}_N\,). (25)$$

5 Simulation

We simulate in Discrete-Frequency domain, extending the method presented in [12]: We generate complex fading amplitudes of a Rayleigh channel with Doppler and delay spread from an i.i.d vector of two complex Gaussian random variables $\boldsymbol{G}$ and $\boldsymbol{G}'$, both with unity variance and length N. The vector $\boldsymbol{G}$ was then multiplied by a precomputed N-by-N matrix $\boldsymbol{U}$, such that $\boldsymbol{U}\boldsymbol{U}^{\mathrm{H}}$ is the channel covariance matrix Γ with elements $\Gamma_{n,m} = \mathrm{E} v_n^{(0)} v_m^{*(0)}$, to create $\boldsymbol{V} = \boldsymbol{U}\,\boldsymbol{G}$. Similarly the derivatives are generated from $\boldsymbol{G}'$ according to $\boldsymbol{V}' = 2\pi f_\Delta T\, \boldsymbol{U}\,\boldsymbol{G}'$.

We simulated an HIPERLAN II type of system under extreme Doppler conditions. A carrier frequency of f_c = 17 GHz, 200 km/h, N = 64 subcarriers, transmit bandwidth of 2 MHz (T = 0.5 μs) and a delay spread of T_{RMS} = 1 μs. E_N / N_0 = 100 (20 dB). The Doppler spread is f_Δ = 3.148 kHz and f_s = 31.25 kHz. So the average SIR for a conventional OFDM system would be around 18 dB; See eq. (22). These parameters correspond to the level of ICI experienced in an 8k DVB-T system at normal vehicle speeds. Figure 2 plots an example of the signal amplitude $|\boldsymbol{V}+\zeta_0\boldsymbol{V}'T\,|$ and $|\boldsymbol{V}'T|$ as a function of the subcarrier number. Figure 3 gives the SINR after the equalization filter for a conventional OFDM system and our proposed solution.

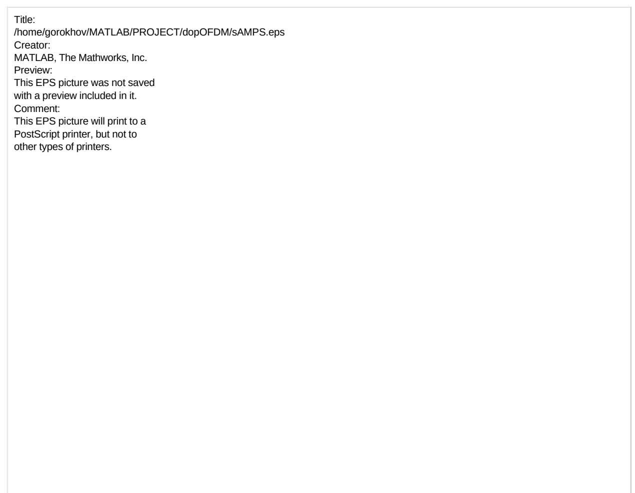
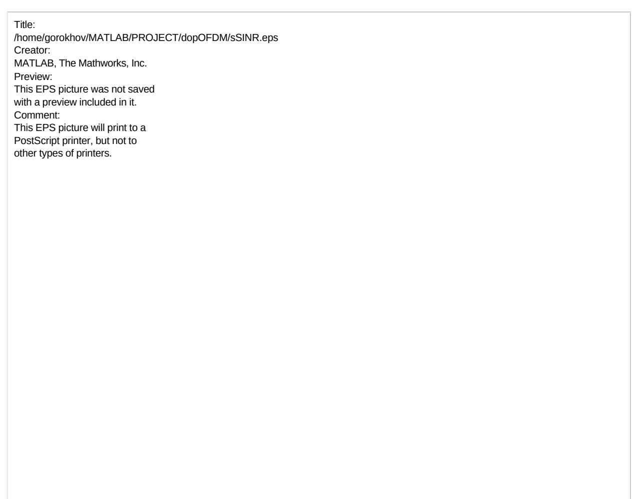


Figure 2: Amplitudes $|\boldsymbol{V}+\zeta_0\boldsymbol{V}'T|$ and derivatives $|\boldsymbol{V}'T|$, as a function of subcarrier number for a sample channel



Figures 3: (ð) Signal-to-ICI-plus-noise ratio in conventional system, (◊) SNIR in new system as a function of subcarrier number for the channel in Figure 2.

Figure 4 aggregates results for 1000 channels versus for every SNR. For low values of the AWGN, the SINR of the conventional system levels off at a value predicted in section 4, whereas our MMSE (Doppler resistant) receiver cancels the ICI.

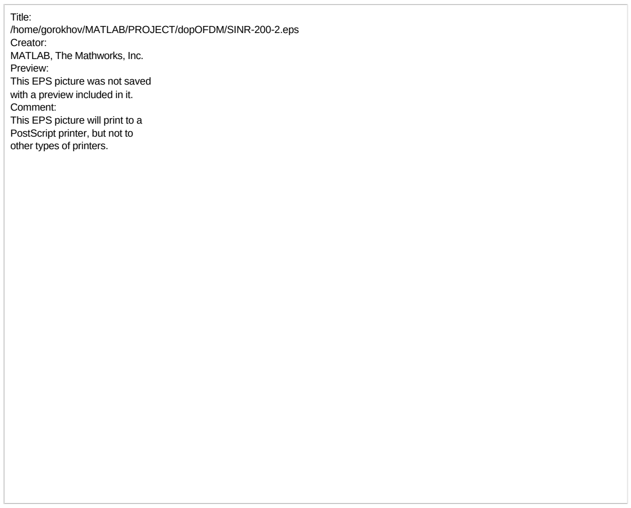


Figure 4: SINR before the slicer, versus input SNR for conventional OFDM receiver and our newly proposed system.

6 Conclusions

Doppler spread is well recognized as a major problem in mobile OFDM reception. Previously contermeasure to mitigate this problem have been searched mainly by considering (i.e., estimating and separating) individual resolvable frequency-shifted components. Estimation, separation and cancellation of these components can become a computationally intensive task, and the effect of signal recovery is sensitive to estimation errors.

This paper took a different approach by modeling the time-varying subcarrier amplitude of each OFDM subcarrier channel as a Taylor expansion with amplitude and derivatives. We showed that this approach is an appropriate model for the Doppler spectrum with (infinitely) many frequency-shifted components. In particular, we argued that all derivatives are complex Gaussian, and we derived the co-variance matrix of the derivatives.

Our approach based on derivatives can be used to design new OFDM receivers that are more robust against Doppler. This simplifies the computational burden on the receiver as it limits the number of channel parameters to be estimated. In the receiver evaluated here, we see that the ICI is largely eliminated.

References

1. S.B. Weinstein and P.M. Ebert, “Data Transmission by Frequency Division Multiplexing Using the Discrete Fourier Transform," IEEE Tans. Commun. Technol., Vol. COM-19, No.5, pp. 628-634, Oct. 1971.
2. L. Cimini, Jr. “Analysis and Simulation of a Digital Mobile Channel Using Orthogonal Frequency Division Multiplexing," IEEE Trans. Commun., Vol. COM-33, No.7, pp. 665-675, July 1985.
3. M. Alard and R. Lassalle, “Principles of Modulation and Channel Coding for Digital Broadcasting for Mobile Receivers," EBU Technical Review, No. 224, pp.168-190, Aug. 1987.
4. J.P. Linnartz and S. Hara, Special Issue on Multi-Carrier Communications, Wireless Personal Communications, Vol. 2 No. 1 & 2, 1995, pp. 1-7.
5. B. Le Floch, R. Halbert-Lassalle and D. Castelain, “Digital Sound Broadcasting to Mobile Receivers," IEEE Trans. Consumer Electronics., Vol. 73, No.4, pp.30-34, Aug. 1989.
6. Wireless Communication, The Interactive Multimedia CD-ROM, 3rd edition, Kluwer Academic Publ. http://www.baltzer.nl/wicom
7. P. Robertson, S. Kaiser, “Effects of Doppler Spreads in OFDM(A) Mobile Radio Systems” VTC Fall ’99, Amsterdam.
8. M. Russel and G. Stuber, “Terrestrial digital video broadcasting for mobile reception using OFDM”, Special Issue on MCC, Wireless Personal Communication, Vol. 2, No., 1&2, 1995, pp. 45-66.
9. W.C. Jakes, "Microwave Mobile Communications", John Wiley and Sons, New York, 1978. New edition 1994.
10. R.H. Clarke, “A statistical theory of mobile radio reception”, Bell Syst. Tech. J., Vol. 47, 1968, pp. 957-1000.
11. T. Aulin, “A modified model for the fading signal at a mobile radio channel”, IEEE Tr. on Veh. Tech., Vol. VT-28, 1979, No. 3, pp. 182-203.
12. N. Yee and J.P.M.G. Linnartz, "Wiener filtering for Multi-Carrier CDMA", conference on Personal Indoor Mobile Radio Communications (PIMRC), The Hague, September 19-23, 1994, Vol. 4, pp. 1344-1347.
13. J.P. Linnartz, “Performance analysis of MC-CDMA in Rayleigh channel with Doppler and delay spread”, IEEE Benelux Symp. on DSP, Hilvarenbeek, 2000.
14. G. Fettweis et al.,"On multi-Carrier CDMA modem design", Proc. of IEEE VTC'94, pp.1665-1669, Stockholm, Sweden, June 1994.
15. http://www.eecs.berkeley.edu/~linnartz/issue.html
16. J.P.M.G. Linnartz, "Performance analysis of synchronous MC-CDMA in mobile Rayleigh channel with both delay and Doppler spreads", submitted in 2000.