

ABOUT THE CAPACITY OF A *DS SPREAD SPECTRUM CDMA* CELLULAR SYSTEM

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Abstract I explain and give an expression for the cell capacity of the uplink in the ideal case of perfect automatic power control. I give the reasons why we need strong power control, its relation to the near-far problem and the cell capacity. I finally give a second expression for the cell capacity of the uplink where I consider the bandwidth efficiency factor, the voice activity factor, sectorization and express the imperfect automatic power control by a factor c_p . I also discuss about adaptive automatic power control used to fight the power control error with a compensation factor.

In Direct Sequence Spread Spectrum *CDMA* Cellular, each user is provided with an individual and distinctive spread spectrum pseudo-noise (*PN*) code. As in all *direct spreading* techniques, the narrow band message signal is multiplied by the spread spectrum pseudo-noise *PN* and then transmitted. Because *PN* codes are uncorrelated with each other, many users can transmit at the same time and in the same radio bandwidth, then we have universal **frequency reuse**. This has the consequence that there is no need for a mobile to change its frequency when moving into another cell, so we have also **Soft Handoff**.

Let's remember that by perfect *APC* (*Automatic Power Control*), we assume that we have the ideal case where the signals received at the base station of a cell from all the mobile units within remains at the same power level, independent of the movement, propagation path loss and location of the mobiles. So by this fact we eliminate the near-far interference problem. This ideal case gives us an optimum for the capacity of a *CDMA* cellular system. This capacity can be greater than other multiple access techniques because contrary to them, *CDMA* users have the same radio bandwidth at the same time, this causing unfortunately **high cochannel interference** which must be fought by **strong power control**.

Before entering in details, let's say that we are in this work concerned only by the capacity of the uplink (mobile to base) called also reverse link. **We keep our attention to the uplink because it is the one which is the most affected by the near-far effect**, a mobile user which tries to reach the base station sees other users as sources of interferences (since all use the same bandwidth j); the signal from the nearest mobile to the base dominates the ones of others. Then in the uplink, the near-far problem is important because too many interferers are involved contrary to the downlink (base to mobile) where the base station because of its fixed and elevated position has access with less opposition to a mobile. So, we **need strong power control in the uplink if we want to have the great capacity that *CDMA* can offer us**.

We will discuss the case of imperfect *APC* later, we firstly consider that we are in a perfect *APC* case and in multiple cells system in which we choose a cell and a mobile user of reference.

In the chosen cell containing N_u mobile users, then N_u being the capacity we are looking for, the number of effective interferers is N_u-1 regardless of how they are distributed in the cell because we consider perfect *APC*. It is important to precise that we assume, firstly the well known classic assumption that the thermal noise is gaussian and secondly that, we are in the case where the N_u-1 users interference total signal appear at the base station as approximately gaussian. From the central limit theorem and the theory of summation of probabilities, it can be shown that this affirmation becomes true when the number of involved distributions increases. We conclude that the second assumption is true for a large number of users in the cell, we need it to avoid useless complicate stochastic models.

In the cell and from a user of reference with our assumption, at the base station the signal-to-interference ratio is :

$$\frac{S}{I} = \frac{\text{Signal}}{\text{All interferences}} = \frac{\text{Signal}}{\text{In-cell interference} + \text{Out-of-cell interference}} \quad (1)$$

, *CDMA* using the same frequency band for each cell, there is interferences coming from the cell of reference (from the N_u-1 users) and the one coming from the neighbouring cells. With perfect *APC*, the base station of reference receive a desired signal of power S and N_u-1 interfering signals, each with the same power S . Then we have :

$$\text{In-cell interference} = (N_u - 1) S \quad (2)$$

Nothing that generally in actual *CDMA* cellular systems, a particular base station is unable to control the power of users in neighbouring cells, perfect *APC* is true only for in-cell users. This because it can be shown² that it is useless to consider interferences from far cells (more than five base station from the one of reference) because they have a negligible effect, only adjacent or near neighbourhood² cells to the one of reference must be considered. If N_{ui} is the number of users in the i^{th} adjacent or near neighbourhood cell and I_i , is the average interference power from a user in the same cell; the out-of-cell interference coming from these total N cells can be written :

$$\text{Out-of-cell interference} = \sum_{i=1}^N N_{ui} I_{ui} \quad (3)$$

Then from (1),(2) and (3), the signal-to-interference ratio becomes :

$$\frac{S}{I} = \frac{S}{(N_u - 1)S + \sum_{i=1}^N N_{ui} I_{ui}} = \frac{S}{(N_u - 1)S \left[1 + \frac{\sum_{i=1}^N N_{ui} I_{ui}}{(N_u - 1)S} \right]} \quad (4)$$

. We can now introduce the frequency reuse efficiency factor of a CDMA system (called also spillover factor) which is the amount of out-of-cell interference compare to the in-cell interference. Then, N_f the frequency reuse efficiency (spillover factor) of CDMA is from (2) and (3 j) :

$$N_f = \frac{\text{Out - of - cell interference}}{\text{In - cell interference}} = \frac{\sum_{i=1}^N N_{ui} I_{ui}}{(N_u - 1)S} \quad (5)$$

Finally putting (5 j in (4) we obtain the S I ratio as:

$$\frac{S}{I} = \frac{1}{(N_u - 1)(1 + N_f)} \quad (6)$$

The expression (6) gives the relation between the capacity N_u and the received signal-to-interference ratio at the base station from the mobile of reference. This formula is not an interesting one because we deal here only in terms of powers without further details. A better way will be to know the relation between the capacity and the modulation or the coding schem, this will have the advantage that besides playing only on signals powers, we can play on modulation or on coding scheme to increase the capacity. The bit energy-to-interference ratio $E_b I_o$, where E_b is the energy per hit and I_o , the interference power spectral density is a well known parameter which depends on modulation and on coding scheme. So, if we know the relation between $E_b I_o$ and $S I$, by using (6) we can find the one between $E_b I_o$ and the capacity N_u .

If R is the baseband information plus overhead bit rate and B_s the system bandwidth., remembering that CDMA uses the entire bandwidth, the $S I$ ratio can be written as :

$$\frac{S}{I} = \frac{R \cdot E_b}{B_s \cdot I_o} = \frac{E_b}{I_o} \times \frac{1}{\frac{B_s}{R}} \quad (7)$$

, where $B_s R$ is known as the processing gain. Then from (7) and (6) we have:

$$\frac{E_b}{I_o} = \frac{B_s}{R} \times \frac{1}{(N_u - 1)(1 + N_f)} \quad (8)$$

which is the relation we were looking for between the capacity N_u and the bit energy-to-interference ratio $E_b I_o$. This expression (8) consider only the Multiple Access Interference (MAI), the one created by the others *CDMA* users simultaneously occupying the same frequency band. If we want to consider the background thermal noise η , we must remember that it has to be added to the interference. Then, if $E_b (I_o + \eta_o)$ is the bit energy-to-interference plus noise ratio (η_o being the noise power spectral density) we have because of (8):

$$\frac{E_b}{I_o + \eta_o} = \frac{1}{\left(\frac{I_o}{E_b} + \frac{\eta_o}{E_b} \right)} = \frac{1}{\frac{R(N_u - 1)(1 + N_f)}{B_s} + \frac{\eta_o}{E_b}} = \frac{1}{\frac{R}{B_s} \left[(N_u - 1)(1 + N_f) + \frac{\eta_o B_s}{E_b R} \right]} \quad (9)$$

Knowing that $\eta_o B_s$ and $E_b R$ equal respectively the noise power η and the signal power S , we can extract N_u from (9) and find the equation :

$$N_u = \frac{B_s R}{\left(\frac{E_b}{I_o + \eta_o} \right) (1 + N_f)} = 1 + \frac{\eta}{S (1 + N_f)} \quad (10)$$

As we can see, (9) and (8) give us different formulas from which we can find the capacity N_u , the difference being that in (9) we have an additional parameter ηS for the background thermal noise. Let's remember that **(10) gives us the expression of the capacity in the case of perfect APC**. The attitude of adding new parameters will be our in the rest of this demonstration to find a better formula for I_t ,,. From (10), we **can already observe that the capacity of a multiple cells system (N_f different from zero) is inferior to the one of the single cell system (N_f equalling zero)** and for a fixed processing gain $B_s R$, we can increase the capacity of the *DS-CDMA* system by reducing $E_b (I_o + \eta_o)$ in using powerful modulation and coding scheme. Because firstly this will have the consequence of increasing the system complexity and secondly, even with powerful techniques it is impossible to reduce $E_b (I_o + \eta_o)$ indefinitely, the only other way to increase N_u is to reduce the interference trough ηS or trough N_f .

So, **the reduction of interferences will be the goal to reach for increasing the *CDMA* system capacity**. The other additional parameters since (10) will be related to the interference so that by acting correctly on them we can modify the capacity and increase it. These additional parameters will be, because of their natures, additional factors in the numerator or in the denominator depending on the fact that they increase or decrease proportionally the capacity of the system. This is the **way** how we will build the most exhaustive parametric model for the capacity.

The additional parameters to be included as factors in a complete formulation of the *CDMA* capacity are :

- N_B : the bandwidth efficiency factor (< 1) because the individual users do not take necessary the whole frequency band available to the system.
- v_f : the voice activity factor (< 1) because of the talklisten nature of a real full-duplex conversation. Remembering that the *CDMA* capacity is interferences limited, a small voice activity factor will give a greater capacity because interferences is reduced proportionally during the non-activity (silent) period. So v_f will be a factor in the denominator of N_u .
- K : the numbers of sectors in the cell-site antenna. The use of sectorization and directional antennas increase the capacity because it reduces interferences ; for example, a cell site with three directional antennas, each having a perfect beam width of 120° , has interference I_o which is one-third of the interference in the case of an omni-directional antenna.
- c_p : the capacity degradation factor (< 1) because of imperfect APC. Because the performance of a *CDMA* system is interferences limited, we need accurate power control. A increasing power control error will reduce the capacity. The value of the power control error will depends on the system implementation and on the algorithm used in the control. We will give later more information about this important and specific characteristic of *CDMA* systems.

Considering all of these new additional factors, we can write a new and complete parametric formula for the capacity of a *CDMA* cellular system. Because they act on the total interference (MAI and noise), they will modify $E_b(I_o + \eta_o)$; if $E_b(I_o + \eta_o)_n$ is the *new* bit energy-to-interference plus noise ratio which takes in consideration our additional parameters, as for (9) we can write :

$$\left(\frac{E_b}{I_o + \eta_o} \right)_n = \frac{N_B K c_p}{v_f} \times \frac{1}{\frac{R}{B_s} \left[(N_u - 1)(1 + N_f) + \frac{\eta}{S} \right]} \quad (11)$$

, then we extract the final complete parametric formula for the capacity :

$$N_u = \frac{N_B K c_p}{v_f} \times \frac{B_s R}{\left(\frac{E_b}{I_o + \eta_o} \right)_n (1 + N_f)} - 1 - \frac{\eta}{S(1 + N_f)} \quad (12)$$

. Formula (12) gives us the complete and final expression we were looking for. This parametric formula have parameters directly related to the interference and it is easy to give them a value in a standard and simple case ; otherwise we need to precise concepts as : diversity, parameters of the coding, . . which ask high stochastic models. So we **conclude** that **(12) is the final parametric formula for the capacity where the imperfect APC is represented by c_p .**

The capacity degradation factor c_p is due to the power control error. So it depends on the system implementation and on the algorithm used in the power control. In [5], we have for : a hexagonal multiple cells system, a processing gain of 255 Hz.s/bits, no sectorization ($K=1$), no voice activity ($v_f=1$), the bandwidth used in totality ($N_B=1$), $(E_b I_o)_n=7\text{dB}$, in cell interference 10 dB upper than the thermal noise and in totality 90 outer interfering cells (here considered total near neighbourhood) with $N_f=0.326$; we have a reduction of 60% for the capacity with 1 dB of power control error. The power control error is interpreted as the standard deviation around a desired power level. Also in this simulation, the path loss exponent was $\beta=4$ and the received power from each mobile user behave as a *log-normal* distribution with zero logarithm mean and a standard deviation measuring the power control error.

It is well known that the value of the bit energy-to-interference plus noise ratio depends on the acceptable bit error rate (a measure of the quality of services : an increasing bit error rate reduce the quality of services); the latest is related to the modulation or the coding scheme used. A increasing (decreasing) value of the bit energy-to-interference plus noise ratio gives a decreasing (respectively increasing) bit error rate. So in observing (12), **we conclude that by increasing the required** quality of services we reduce the possible allowable capacity. In digital voice transmission, a bit error rate of 10^{-3} gives a acceptable quality for the received voice message. Let's say that N_f is related to the quality of macroscopic divers& and the most optimistic assumption is $N_f < 0.5$. The voice activity factor v_f is usually near 0.4 in a standard conversation.

As we see, the value of capacity degradation factor is the most difficult to find because it is related to power control. Power control is capital in *CDMA* because interferences limits the capacity. The system capacity is maximised if each mobile transmitter power level is controlled so that its signal arrives at the base station with the minimum required signal-to-interference ratio. If a received signal is too weak, we will be under the required minimum for the concerned user. If this signal is too great, we will be correct for that user but this will add undesired interference to all other users because in *CDMA* they share the same frequency band. A received signal will be too great or too weak depending on whether a mobile user is near or far from the base station because during the travel there are channel fluctuations which cause **shadowings** and **fadings**. We see that to solve this near-far problem we need a compensation in case of shadowing and fading to fight the power control error they create. Adaptive power control is used in *CDMA* Spread Spectrum systems to compensate the near-far problem and to increase the capacity.

The more is the quality of the adaptive power control, the more accurate and precise will be the compensation factor, this fact will reduce properly the power control error (giving a better *APC*) and increase the capacity. There is two distinct categories of power controls : **open-loop** and **closed-loop**. In open loop power control, only the **downlink** (base to mobile) is used to compensate for the near-far-interference problem. in close **loop** power control, we use both the **downlink** and the **uplink** to have a more precise compensation. **We can conclude that because of time delays, closed-loop adaptive power control is inadequate for satellite communications.** The value of the compensation factor and its relation to power control error depends on the algorithm used. We give in the following lines a simple **model from [6] and [7]** used to show a **possible** way how power control compensation can work.

We consider an open loop power control algorithm where a user of reference is considered shadowed during a period of time A and not shadowed during $1-A$. During A , the channel (frequency-nonselective) is modelised by a *Raleigh* random variable and during $1-A$ while the user is not shadowed by a *Rice* random variable. A *Rice* channel modelised a direct path (specular component) plus a multiple path (diffuse or scattered component) but, in the *Raleigh* one only a multiple path (diffuse or scattered component) is considered, that is why it can represent the statistics of the channel while the user of reference is shadowed. From the theory of random variables, without power control, the probability density function of the received signal ρ at the base station is given by [6]:

$$f(\rho) = A \frac{\rho}{\sigma^2} e^{-\frac{\rho^2}{2\sigma^2}} + (1-A) \frac{\rho}{\sigma^2} e^{-\left(\frac{\rho^2 + A_s^2}{2\sigma^2}\right)} I_0\left(\frac{\rho A_s}{\sigma^2}\right) \quad (13)$$

where A_s is the amplitude of the specular component of the rician part of the density and $2\sigma^2$ is the average power in the scatter component of the fade, and finally $I_0(x)$ is the modified *Bessel* function of the first kind and zeroth order. We define the *Rice* factor as:

$$c = \frac{A_s^2}{(2\sigma^2)} \quad (14)$$

If we normalise to $A_s=1$, we have about the powers [7]:

- Average received power while the user is shadowed :

$$P_s = 2\sigma^2 \quad (15)$$

- Average received power while the user is unshadowed :

$$P_u = 2\sigma^2(1+c) \quad (16)$$

We see that without power control, the average received power while the user is shadowed is attenuated by a factor $F=1/(1+c)$ compared to the unshadowed case. The goal of the power control here is to compensate by a factor P for the loss of power due to shadowing. Then, the received power during the shadowed period becomes [7]:

$$P_s = 2\sigma^2(1+c)P \quad (17)$$

, so that with $P=1$ there is no compensation, the power received during the shadowed period being the same as during the unshadowed period (16), giving a perfect *APC* case.

The correction brought by P against the power control error will allow a better (accurate) received signal. References [7] and [8] give the complex expression of the conjoint probability density function of the amplitude of the received signal and the power control error.

Before finishing, let's say that because users share the same radio frequency band at the same time, the capacity of the *DS Spread Spectrum CDMA* can be much greater than other multiple access technologies, but is limited by interferences. If *TDMA* and *FDMA* are bandwidth limited, in *CDMA* we need strong power control to solve the interferences problems and at the same time take the advantage of the possible great capacity. In our days, with the advances in signal processing, electronic knowledges in waves propagation, we can have accurate power control techniques. This has helped the actual growth of *CDMA*.

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