Narrowband Multipath Fading:

A Simple Model

Doppler in Wireless Channels

Scenario: mobile at position x, moving with velocity V, with signal arriving at angle θ.

With a nominal carrier frequency of f_0, the received signal is

$$r(t) = Ae^{j(2\pi f_0 t - \beta x \cos\theta)}$$

with phase (where $\beta = \frac{2\pi}{\lambda}$ = wave number)

$$\Phi(t) = 2\pi f_0 t - \beta V t \cos\theta$$

and resulting Doppler frequency

$$f_D = \frac{V}{\lambda}\cos\theta$$

Another way: from physics, for a radial (approaching) velocity V_r,

$$f_D = f_0 \frac{1}{\frac{c}{V_r} - 1} = \frac{f_0 V_r}{c - V_r} \approx \frac{f_0 V_r}{c}$$

But $V_r = V \cos\theta$ and $c = f_0 \lambda$ so

$$f_D = \frac{V}{\lambda} \cos\theta$$

Often, V and θ will vary with time, so f_D will vary correspondingly, causing *Doppler spread.* Often this is modeled statistically. In general, the Doppler spread characterizes the rate of channel variations.

Amplitude Variation Due to Motion

Now consider the case where two incoming waves arrive at the mobile at angles of 0 and θ. Assume that they are of equal amplitude. Then, from above, the received signal is

$$r(t) = Ae^{j2\pi f_0 t}(e^{-j\alpha} + e^{-j\alpha \cos\theta})$$

where $\alpha = \beta x$. This can be expressed as

$$r(t) = Ae^{j2\pi f_0 t}(e^{-j\alpha(U+D)} + e^{-j\alpha(U-D)})$$

where

$$U = \frac{1+\cos\theta}{2}, D = \frac{1-\cos\theta}{2}$$

Factoring, we have

$$r(t) = Ae^{j2\pi f_0 t}e^{-j\alpha U}(e^{j\alpha D} + e^{-j\alpha D})$$

Notice that

$$e^{j\alpha D} + e^{-j\alpha D} = 2\cos\left[\frac{\alpha(1-\cos\theta)}{2}\right],$$

so

$$r(t) = Ae^{j2\pi f_0 t} e^{-j\beta x(1+\cos\theta)/2} \cdot 2\cos\left[\frac{\beta x(1-\cos\theta)}{2}\right]$$

Thus the phase of signal due to Doppler is

$$\Phi(t) = \frac{1}{2}\beta x(1+\cos\theta)$$

and the Doppler frequency is

$$f_D = \frac{1}{2\pi}\frac{1}{2}\frac{2\pi}{\lambda}V(1+\cos\theta) = \frac{V}{2\lambda}(1+\cos\theta)$$

But the signal amplitude fluctuates due to multipath according to

$$\Theta_{SW}(t) = \frac{\beta x}{2}(1 - \cos\theta),$$

i.e., with frequency

$$f_{SW} = \frac{V}{2\lambda}(1 - \cos\theta).$$

Examples: If $\theta = 0$ then both waves are arriving from the same direction, and $f_{SW} = 0$. If $\theta = \pi$ (opposite directions) $f_{SW} = \frac{V}{\lambda}$.

Interpretation: as mobile moves with constant velocity in a fixed multipath environment, amplitude of the received signal fluctuates with frequency f_{SW}. This is sometimes called the "amplitude standing wave pattern" due to motion. Notice that as V increases, so does f_{SW}.

Remarks

This scenario illustrates two key properties of wireless multipath channels:

1. The *fade level* (amplitude) varies as a function of *position.* In the general case when a large and random number of paths exist, the resulting amplitude (envelope) is modeled as a random variable (e.g., a Rayleigh r.v.).
2. The *rate* of fading fluctuations depends on *velocity.* A rough measure of the fading rate is the Doppler spread.

Finally, note that channel variations (both level and rate of change) can be caused by objects (i.e., those that reflect or absorb the transmitted energy) other than the transmitter and receiver.