Direct-Sequence Spread Spectrum

Consider a binary discrete-time communication system with the received signal a time sequence $\{r_m\}_{m=0}^{\infty}$ defined by

$$r_m = \mathcal{E} b_m + w_m$$

where b_m is the sequence of information symbols (antipodal binary, $b_m \in \{\pm 1\}$) and w_m is additive zero-mean white Gaussian noise (AWGN), i.e.,

$$E[w_m] = 0, \qquad E[w_m w_{m+l}] = \sigma^2 \delta(l).$$

We use a correlation receiver to determine whether a $+1$ or a -1 was transmitted at a time instant m. We assume that the transmitter is connected to an information source which outputs $+1$'s and -1's with equal probability. In this case, such a receiver is a simple level detector:

$$r_m \geq 0 : \text{decide that } +1 \text{ was sent}$$

$$r_m < 0 : \text{decide that } -1 \text{ was sent}$$

Here $r_m = y_m$ is known as the decision variable. Its statistics determine the performance of the receiver. It is easy to see that it is a normal (Gaussian) random variable with mean $\mathcal{E}b_m$ and variance σ^2.

Now consider modulating each symbol with another ± 1-valued sequence (a spreading sequence) $\{c_n\}_{n=0}^{n=N-1}$ such that each symbol b_m results in the transmission of

$$\text{either } \ c_0, c_1, ..., c_{N-1} \ \text{ or } \ -c_0, -c_1, ..., -c_{N-1},$$

depending on the value of b_m.

Thus each bit of duration T is coded into a sequence of N **chips** of duration $T_c = T/N$. The increase in signaling rate **spreads the spectrum** of the transmitted signal by a factor of N.

The received sequence can be described as (dropping the subscript m for clarity)

$$r_n = \mathcal{E}_c b c_n + w_n, \quad n = 0, 1, ..., N-1.$$

where $\mathcal{E}_c = \frac{\mathcal{E}}{N}$ and $E[w_n^2] = \frac{\sigma^2}{N}$.

Now let us specify two properties of the spreading sequence $\{c_n\}$: it has a mean value of approximately zero, i.e.,

$$\sum_{n=0}^{N-1} c_n \approx 0,$$

and an autocorrelation given by

$$\sum_{n=0}^{N-1} c_n c_{n+i} \approx \begin{cases} N, & i = 0 \\ 0, & \text{otherwise.} \end{cases}$$

These conditions are ideal, but can be closely approached in practice. These properties give sequences of this type a noise-like appearance; thus the name pseudo-noise (PN) sequences.

Then, the correlation receiver performs the following operation to obtain the decision variable y:

$$y = \sum_{n=0}^{N-1} r_n c_n$$

or

$$y = \sum_{n=0}^{N-1} (\mathcal{E}_c b c_n + w_n) c_n,$$

which yields, based on the properties of the spreading sequence,

$$y = N\mathcal{E}_c b + \sum_{n=0}^{N-1} w_n c_n.$$

Hence our decision variable is normal with mean $N\mathcal{E}_c b = \mathcal{E}b$ and variance σ^2. Comparing this result with the non-spread system above shows that spreading yields no improvement in the ideal AWGN channel. This can be seen intuitively by noting that the signaling rate is increased by a factor of N, but this also increases the signal bandwidth, and therefore the noise power, by a factor of N.

As we will see, the power of spreading comes from its effect on narrowband or correlated signals. These include interference, multipath, or signals from other transmitters the network.

Now suppose the channel contains an interferer: an unknown constant is added to the received signal. Then we have

$$r_n = \mathcal{E}_c b c_n + i_n + w_n, \quad n = 0, 1, ..., N-1.$$

where $i_n = I$, a real constant. Then our correlation receiver produces the decision variable

$$y = \sum_{n=0}^{N-1} (\mathcal{E}_c b c_n + i_n + w_n) c_n$$

which becomes

$$y = N\mathcal{E}_c b + I \sum_{n=0}^{N-1} c_n + \sum_{n=0}^{N-1} w_n c_n$$

or

$$y \approx N\mathcal{E}_c b + 0 + \sum_{n=0}^{N-1} w_n c_n,$$

yielding again a decision variable with mean $N\mathcal{E}_c b = \mathcal{E} b$ and variance σ^2, so the interference is suppressed by the despreading (correlation) operation.

In contrast, the decision variable in non-spread system would have a mean of $\mathcal{E} b + I$, which will render the system unusable for $|I|$ sufficiently large.

Remark: Notice that the recovery of the signal requires that the receiver's own copy of the spreading sequence be synchronized with the received version. This is a key requirement in spread spectrum system design.

Note: From now on, we normalize to $\mathcal{E}_c = 1$ (unit chip energy) for simplicity, so that the bit energy is N.

Now consider a multipath channel, with a direct path and a specular (reflected) path which causes another copy of the signal to arrive at a delay of l with unknown attenuation β:

$$r_n = \begin{cases} b_m c_n + \beta b_{m-1} c_{N-l+n}, & n = 0, ...l-1 \\ b_m c_n + \beta b_m c_{n-l}, & n = l, ...N-1, \end{cases}$$

where we assume that $l < N$, i.e., the delay is less than one symbol duration.[a]

Notice the specular path causes interference from both a delayed version of the desired symbol and the previously transmitted symbol b_{m-1}.

[a] This assumption can be relaxed using straightforward methods.

Here,

$$y_m = Nb_m + \beta b_{m-1} \sum_{n=0}^{l-1} c_{N-l+n} c_n + \beta b_m \sum_{n=l}^{N-1} c_{n-l} c_n + \sum_{n=0}^{N-1} w_n c_n,$$

which becomes

$$y_m \approx Nb_m + 0 + 0 + \sum_{n=0}^{N-1} w_n c_n.$$

The multipath signal is suppressed by the despreading. In the case of the unspread system, this channel could cause severe ISI, resulting in a performance loss.

Direct-Sequence Code Division Multiple Access

Now assume there are K users (transmitters), where the kth transmitter modulates its data with the spreading sequence $\{c_n^{(k)}\}$. This set of **signature sequences** or **spreading codes** has the crosscorrelation property

$$\sum_{n=0}^{N-1} c_n^{(k)} c_{n+i}^{(j)} \approx \begin{cases} N, & k = j, i = 0 \\ 0, & \text{otherwise} \end{cases}$$

Thus we have a set of K sequences with zero crosscorrelations and impulse-valued autocorrelations. (Note that this includes the earlier autocorrelation as a special case.)

Assume that all K users simultaneously transmit, and we are interested only in the signal from user $k = 1$. Assuming time synchronization between users, the received signal is

$$r_n = \sum_{k=1}^{K} b_m^{(k)} c_n^{(k)} + w_n$$

It follows that the correlation receiver for user 1 generates the decision variable

$$y_m^{(1)} = b_m^{(1)} \sum_{n=0}^{N-1} (c_n^{(1)})^2 + \sum_{k=2}^{K} b_m^{(k)} \sum_{n=0}^{N-1} c_n^{(k)} c_n^{(1)} + \sum_{n=0}^{N-1} w_n c_n^{(1)}$$

Using the crosscorrelation property,

$$y_m^{(1)} \approx N b_m^{(1)} + 0 + \sum_{n=0}^{N-1} w_n c_n^{(1)}.$$

Therefore, the crosscorrelation property of the sequences allows simultaneous transmissions in the same channel to be successfully detected. This property allows spread spectrum to be used as a multiple-access method (like TDMA or FDMA); it is usually called code-division multiple access, or CDMA.

Note: Normally, the different transmitters would not be time-synchronized, so all signals would be received with different relative delays. However, due to the crosscorrelation property, it is easy to show that the same result is true when the other $K-1$ signals have arbitrary delays relative to the desired signal.

DS Spread Spectrum – Summary

Notice that the multipath and multi-user interferences are additive. Therefore, the previous results can be combined to show the spread spectrum provides resistance combinations of

- multipath
- multi-user interference

Considerations:

- The sequence properties used here are idealized; degradation can occur in practice. Solutions include **coding**, **power control**, and **multi-user receivers**.
- The sequences are periodic; the period may be greater than a symbol duration to prevent performance loss.